Mr. Hansen
STAtistics HW
due 2/26/010

#10.60, 10.62, 10.64, + 1 extra problem

#10.60 Let μ = true mean speaking-out score for males from non-Eng.-speaking Asian countries

$H_0: \mu = 10$

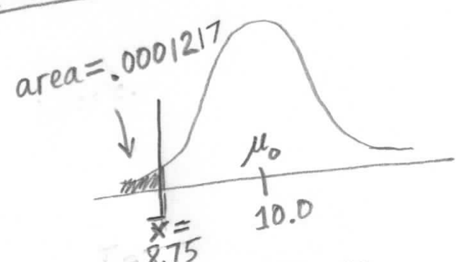
$H_a: \mu < 10$

Assumptions for 1-sample t test:

- SRS? Probably not; "random" sample is stated, but an SRS from the pop. of interest would be essentially impossible to engineer. Proceed with caution.
- Normal pop.? Not required, since $n = 64$ (large sample). The t test is acceptable as long as the pop. does not have extreme skewness or extreme outliers, and with a scale of only 3 to 15 for scores, we should be safe. [The pop. here will not be like the wealth of individuals, where a single outlier like Bill Gates could throw everything off.]

[• σ unknown? ✓]

Sketch of sampling distrib. of $\bar{x}$, assuming H_0 is true:



$$s.e. = \frac{s}{\sqrt{n}} = \frac{2.57}{\sqrt{64}} = .3213$$

[Note: α was not specified in this problem.]

$$t = \frac{\text{obs.} - \text{hyp.}}{s.e.} = \frac{\bar{x} - 10}{\frac{s}{\sqrt{n}}} = \frac{8.75 - 10}{.3213} = -3.890$$

$P = .0001217 \Rightarrow$ reject H_0 at almost any reasonable α level

Conclusion: There is very strong evidence ($t = -3.89$, $df = 63$, $P = .00012$) that the true mean speaking-out score for males from non-Eng.-speaking Asian countries is less than 10.

#10.62 Let μ = true mean time to distraction for teenage boys working on an indep. task

$H_0: \mu = 5$

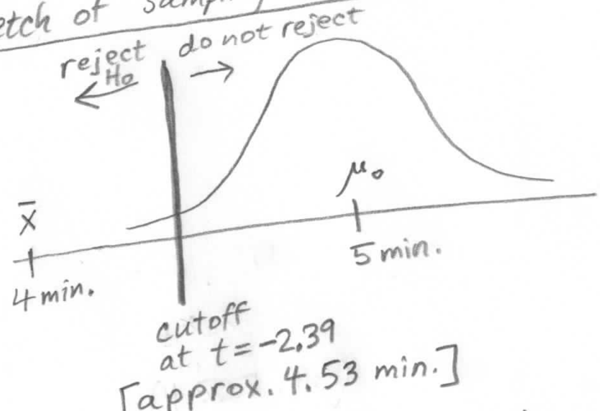
$H_a: \mu < 5$

Assumptions for 1-sample t test:

- SRS? Definitely not met. This was a "random sample" (prob. not an SRS) of "Australian boys." Since cultural factors may play a role in attention span, Australian boys cannot be assumed to be representative of all teenage males. Proceed with caution.
- Normal pop.? Not required, since $n = 50$ (large sample). However, some extreme right skewness in the pop. is possible: imagine a boy who can focus for many hours to prove a point. Proceed with caution.

[• σ unknown? ✓]

Sketch of sampling distrib. of $\bar{x}$, assuming H_0 is true:



$\alpha = .01$ (given)

$s.e. = \frac{s}{\sqrt{n}} = \frac{1.4}{\sqrt{50}} = .19799$

$df = n - 1 = 49$

$t^* = 2.39$ by table

[Note: Since there is no row for 49, we have to use the next larger df line in the table. Use column for area = .98, since that leaves .01 in each tail.]

$$t = \frac{\text{obs.} - \text{hyp.}}{s.e.} = \frac{4-5}{\frac{s}{\sqrt{n}}} = \frac{-1}{.19799} = -5.05$$

$P = .00003249 < .01 \Rightarrow$ reject H_0

Conclusion: If the sample is representative of all teenage males, and if the pop. of attention spans is not excessively skewed, then there is sufficient evidence ($t = -5.05$, $df = 49$, $P < .01$) that the true mean time to distraction for teenage boys is less than 4 minutes. We assume that "time to distraction" and "attention span" are synonymous.

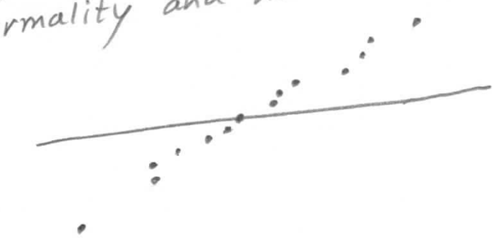
#10.64 Let μ = true mean amino acid uptake rate for cultures grown in presence of nitrates

$H_0: \mu = 8000$

$H_a: \mu < 8000$

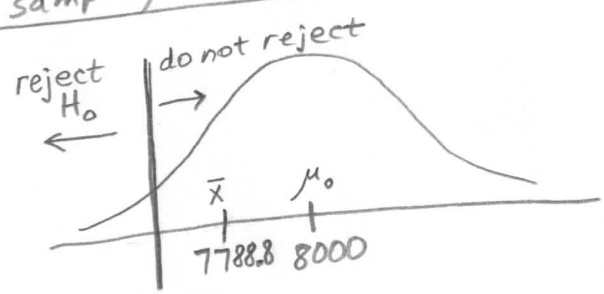
<u>Assumptions for 1-sample t test:</u>

- SRS? ✓ It is customary to treat experimental units as if they were an SRS drawn from all possible experimental units [cultures, in this case] that could have been used.
- Normal pop.? ✓ Reasonable assumption in most problems involving a natural process. NQP of raw data shows no pronounced lack of normality and no outliers:



[• σ unknown? ✓]

<u>Sketch of sampling distrib. of $\bar{x}$, assuming H_0 is true:</u>



cutoff
at $t = -1.35$
[approx. 7650 amino acid uptake rate]

$\alpha = .1$ (given)

$s.e. = \frac{s}{\sqrt{n}} = \frac{1002.43}{\sqrt{15}} = 258.826$

$df = n-1 = 14$

$t^* = 1.35$ by table
[Use row for df=14 and column for 80% area, since that leaves 10% in each tail.]

$$t = \frac{\text{obs.}-\text{hyp.}}{\text{s.e.}} = \frac{\bar{x}-8000}{\frac{s}{\sqrt{n}}} = \frac{7788.8-8000}{258.826} = -.816$$

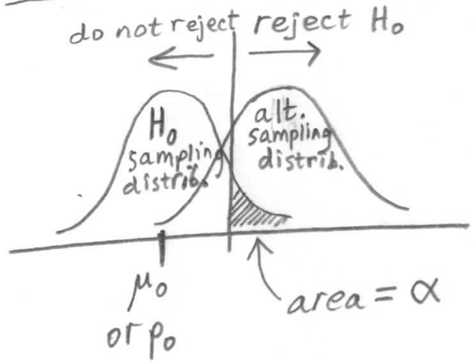
$P = .214 > .1 \Rightarrow$ do not reject H_0

<u>Conclusion:</u> There is insufficient evidence ($t = -.816$, $df=14$, $P>.1$) to conclude that the true mean amino acid uptake rate for cultures grown in the presence of nitrates is less than the value of 8000 given for cultures without nitrates. [In other words, the observed sample mean that is below 8000 can be plausibly explained by chance. The difference is not statistically significant at the $\alpha = .1$ level.]

<u>Final problem.</u> Show that constant 75% power against flexible alternatives leads to an increase in Type I error probability for alternatives that are closer to the null-hypothesis value of the parameter in either a one-tailed or a two-tailed test.

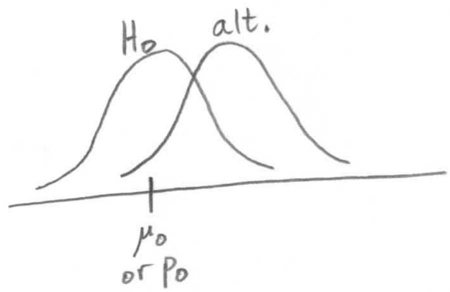
Given: $\beta = .25$ (fixed)

<u>Case I: One-tailed test.</u>

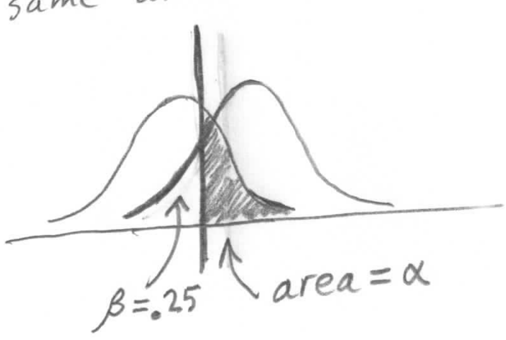


This diagram shows how only a small area (of size α) of the H_0 sampling distribution crosses into the "reject" zone. For the alternative sampling distribution, 75% of the area (i.e., power) lies within the "reject" zone.

Now, consider a different alternative that is closer to μ_0 or p_0:

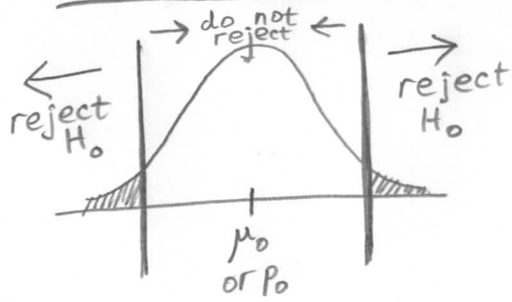


The only way to maintain a fixed power of .75 (i.e., $\beta = .25$) is to move the reject/non-reject boundary in the same direction that we moved the alternative, like this:



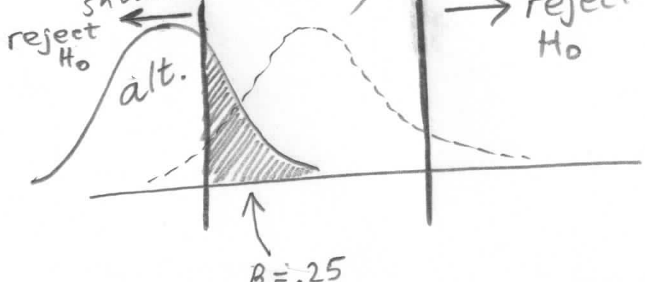
However, by moving the boundary in this way, so as to keep $\beta = .25$, we have necessarily increased α as shown by the shaded area. (Q.E.D.)

Case II: Two-tailed test.

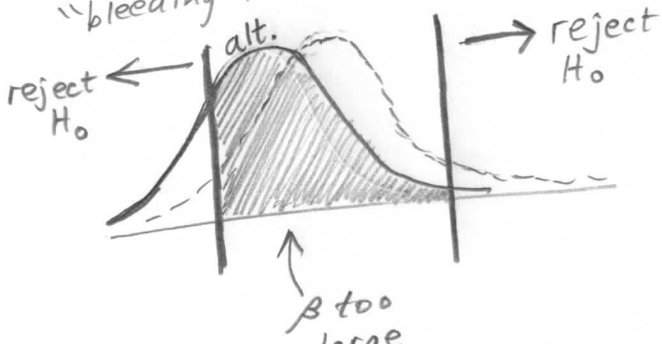


This diagram shows the sampling distribution of $\overline{X}$ or $\hat{p}$, subject to the assumption that H_0 is true. The two shaded areas add up to α.

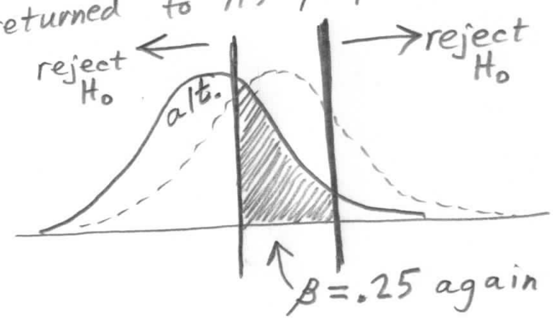
Consider an alternative sampling distribution that exhibits $\beta = .25$. (For clarity, the original H_0 distribution is shown dotted.)



Now, if we select a different alternative that is closer to the hypothesized parameter value (namely, μ_0 or p_0), we will have more of the alternative sampling distrib. "bleeding into" the do-not-reject zone, as shown here:



The only way to counteract this problem of $\beta > .25$ is to pull both boundaries closer in, so that β can be returned to its proper value of .25, like this:



However, now the tails of the original H_0 sampling distribution fall much more in the "reject" zone, meaning that α has increased. (Q.E.D.)