Mr. Hansen
Geom. Sample Problem
"The X"
10/2/2008

<u>Note to students:</u>

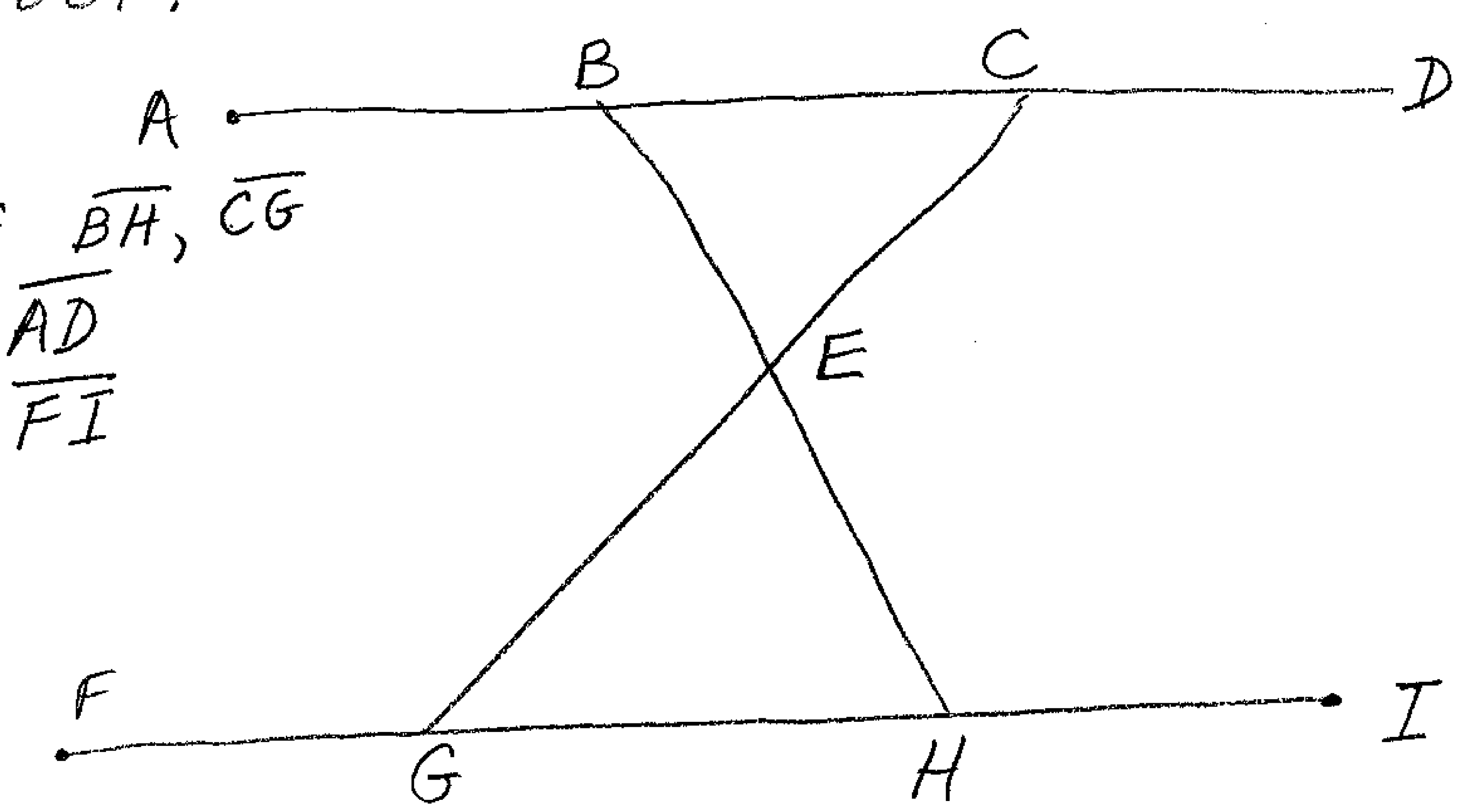
I hope you enjoy this problem. It is an example of how a combination of forward and backward chaining can be used to attack a proof that may be too hard for a "single step" solution. Remember, when I asked how many of you knew how to solve the problem just by looking at it, fewer than half the students in each class raised their hands.

Over the course of the year, we will naturally do many problems that are harder or more intricate than this one. However, this is a good place to start, since it illustrates the two basic principles of general problem solving:

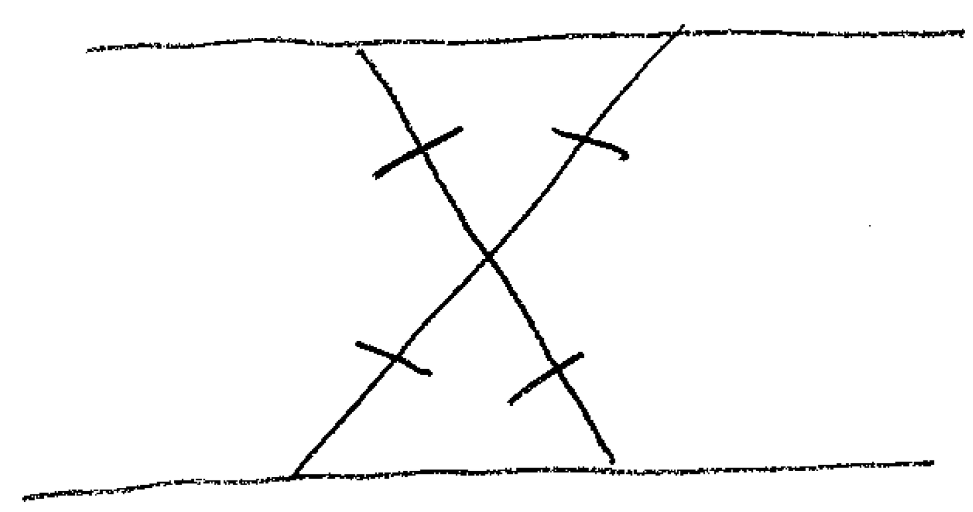
1) Start by doing all the obvious forward chaining that you can do.
2) Then, use backward chaining and see if you can "close the gaps" to make a complete proof.

Given: E is mdpt. of $\overline{BH}$, $\overline{CG}$
B, C trisect $\overline{AD}$
G, H trisect $\overline{FI}$

Prove: $\overline{AD} \cong \overline{FI}$



<u>Step 1</u>: Do as much forward chaining as you can. In other words, mark the givens on the diagram, <u>being careful not to introduce any unwarranted assumptions</u>. For example, you cannot mark the midpoint facts as



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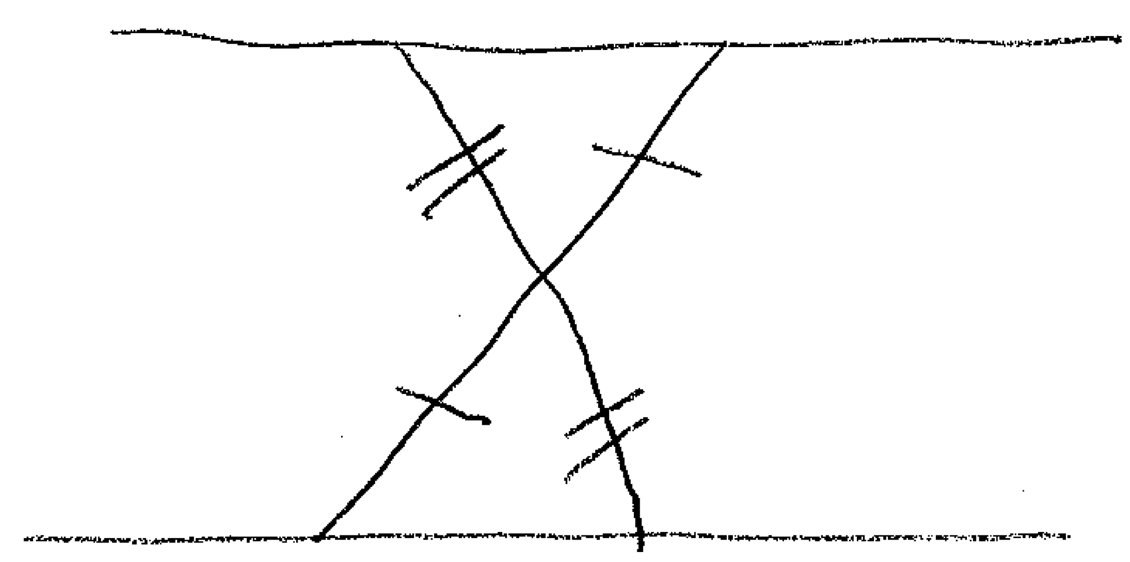
but you <u>can</u> safely mark them like this:



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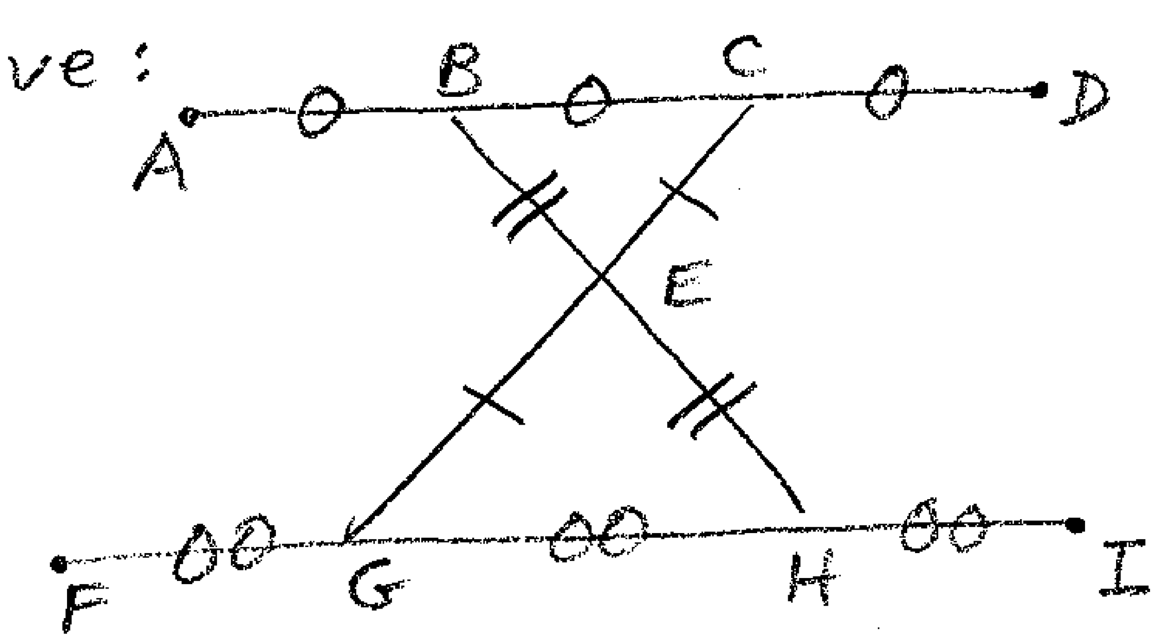
Step 1a: If necessary, redraw the diagram or make a larger one if necessary. Paper is cheap!

Mark remaining givens, again taking care not to assume anything unwarranted. In our example that means that we "forward chain" from the trisection facts to mark groups of congruent segments, but we do <u>not</u> assume that $\overline{AD}$ and $\overline{FI}$ are both split into the same subdivisions. (That would be circular reasoning! Our goal is to show that $\overline{AD} \cong \overline{FI}$. We cannot assume that the segments are already congruent and are trisected equally.)

Step 1b: If there are any obvious consequences (such as congruent Δs), you may wish to note those as well, since they would give other opportunities for forward chaining. However in the example "X" problem, we have probably done about as much forward chaining as we can.

<u>Step 2</u>: Look at the goal of the problem (in our example, $\overline{AD} \cong \overline{FI}$) and see if there is a "pre-goal" which, if true, would make the final goal very easy to obtain. In other words, we want to try some <u>backward</u> chaining now.

Here is what we have:



"Oh, my!" you say to yourself. "If <u>only</u> we knew that $\overline{BC} \cong \overline{GH}$, we could apply the Multiplication Property, and we'd be finished."

OK, then jot down $\overline{BC} \cong \overline{GH}$ as a pre-goal.

But is there some pre-pre-goal that would make $\overline{BC} \cong \overline{GH}$ obvious, or at least an easy problem to solve?

As a matter of fact, there is, "Oh, yes," you say to yourself, "if <u>only</u> we could get the two triangles to be congruent, then $\overline{BC} \cong \overline{GH}$ would be an easy consequence of CPCTC."

OK, so you now jot down Δupper $\cong$ Δlower as a pre-pre-goal. (Don't worry about the letter names of the triangles just yet. Your purpose in jotting this down is simply to remind you of what your pre-pre-goal is.)

Is there some way of proving the triangles to be congruent? "Hmmm," you say, "so far all we know about is ASA, SAS, and SSS." So list (in your mind) what you know:

- ASA does not look promising, since no angle information was given. How are you ever going to get <u>2 pairs</u> of angles, which is what you need?
- SSS looks somewhat promising, but beware! The single blob on $\overline{BC}$ (see diagram on p. 3) is not necessarily the same as the double blob on $\overline{GH}$. Oh, sure, it <u>might</u> be, but we really can't assume that. In fact, check your notes: $\overline{BC} \cong \overline{GH}$ was on the <u>pre-goal</u> list. To

treat $\overline{BC} \cong \overline{GH}$ as a fact at this point would be to commit the fallacy of circular logic or circular reasoning. This happens so often on student papers that Mr. Hansen has a special abbreviation, CIRC, to indicate this type of error.

- That leaves only SAS, by process of elimination. Now look around the triangles. To apply SAS would require yet another pre-pre-pre-goal, namely knowing $\angle BEC \cong \angle HEG$, since that is the only way to get an angle trapped between two sides that match the corresponding sides in the other triangle. Therefore, add $\angle BEC \cong \angle HEG$ to your goal list.

This is what your goal list should look like now:

I WANT $\overline{BC} \cong \overline{GH}$,
which requires Δupper $\cong$ Δlower,
which requires $\angle BEC \cong \angle HEG$.

"Can this last pre-pre-pre-goal be satisfied?" you ask yourself. Why yes! It can. Since $\angle BEC$ and $\angle HEG$ are vertical angles, they have to be congruent. That will cause ΔBEC to be $\cong$ to ΔHEG (we use the letters now), which in turn will make $\overline{BC} \cong \overline{GH}$, which lets us apply the Multiplication Property to finish the proof. Whew! Now, before you turn to the next page, see if you can write the proof by yourself.

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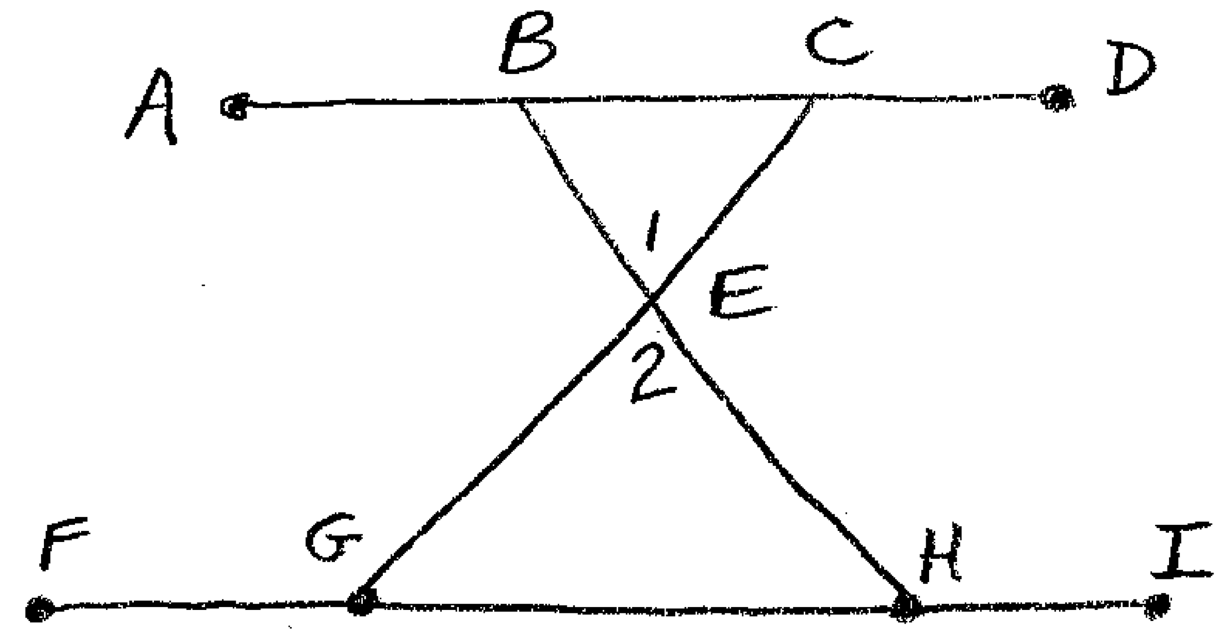

1. E mdpt. of $\overline{BH}$	1. G
2. $\overline{BE} \cong \overline{EH}$	2. Def. mdpt.
3. E mdpt. of $\overline{CG}$	3. G
4. $\overline{CE} \cong \overline{EG}$	4. Def. mdpt.
5. $\angle 1 \cong \angle 2$	5. Vert. $\angle$s
6. $\triangle BEC \cong \triangle HEG$	6. SAS (2, 5, 4)
7. $\overline{BC} \cong \overline{GH}$	7. CPCTC
8. B, C tris. $\overline{AD}$	8. G
9. G, H tris. $\overline{FI}$	9. G
10. $\overline{AD} \cong \overline{FI}$	10. Mult. Prop. (7, 8, 9)

$\square$

Were you able to write the proof? If yes, give yourself a gold star! If not, turn this page over and try again.

REMEMBER THE BAMBOO