

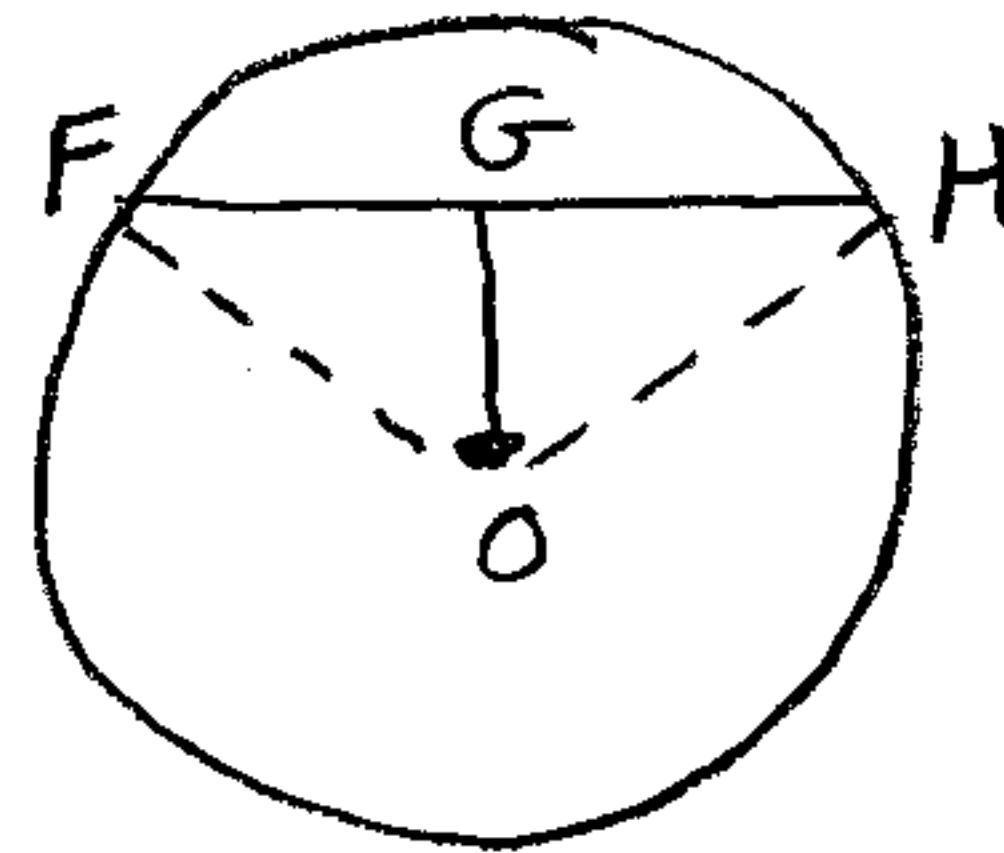
Selected problems, pp. 162-167

pp. 162-164

3. Given: $\odot O$

$$\overline{OG} \perp \overline{FH}$$

Prove: $\overline{FG} \cong \overline{GH}$

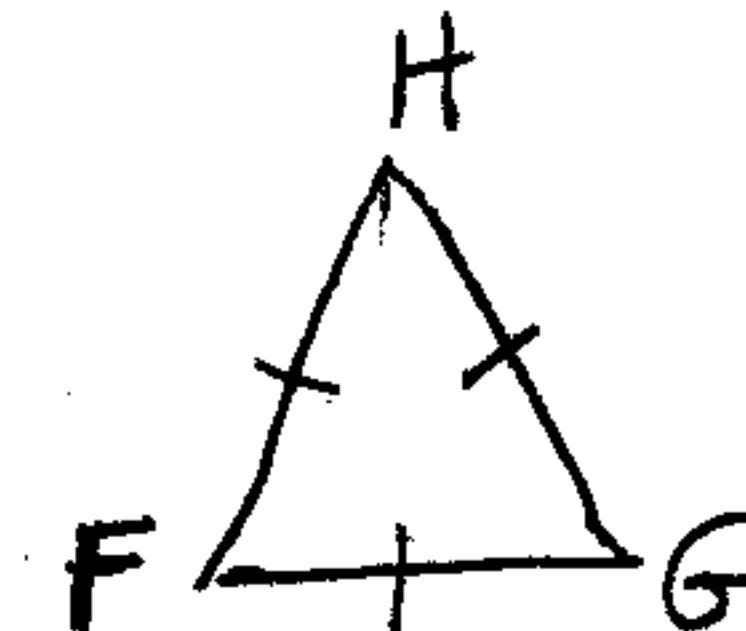


- 1. $\odot O$
- 2. $\overline{OG} \perp \overline{FH}$
- 3. Construct $\overline{OF}, \overline{OH}$
- 4. $\angle OG F, \angle OG H$ are rt.
- 5. $\triangle OGF, \triangle OGH$ are rt.
- 6. $\overline{OF} \cong \overline{OH}$
- 7. $\overline{OG} \cong \overline{OG}$
- 8. $\triangle OGF \cong \triangle OGH$
- 9. $\overline{FG} \cong \overline{GH}$

- 1. G
- 2. G
- 3. Two pts. det. a line
- 4. Def. \perp
- 5. Def. of rt. Δ
- 6. Radii of a \odot are \cong
- 7. Refl.
- 8. HL (5, 6, 7)
- 9. CPCTC

]

7. Given: $\triangle HGF$ is equilat.



a) $m\angle F = m\angle H$ by IIT

$$x + 32 = 2x + 4$$

$$\textcircled{28} = x$$

$$m\angle G = m\angle F = x + 32 = 28 + 32 = \textcircled{60}$$

Note: All 3 angles are 60° since for triangles, equilat. \Leftrightarrow equiangular.

b) perim. = $3HG$

$$6y + 24 = 3(3y - 7)$$

$$6y + 24 = 9y - 21$$

$$45 = 3y$$

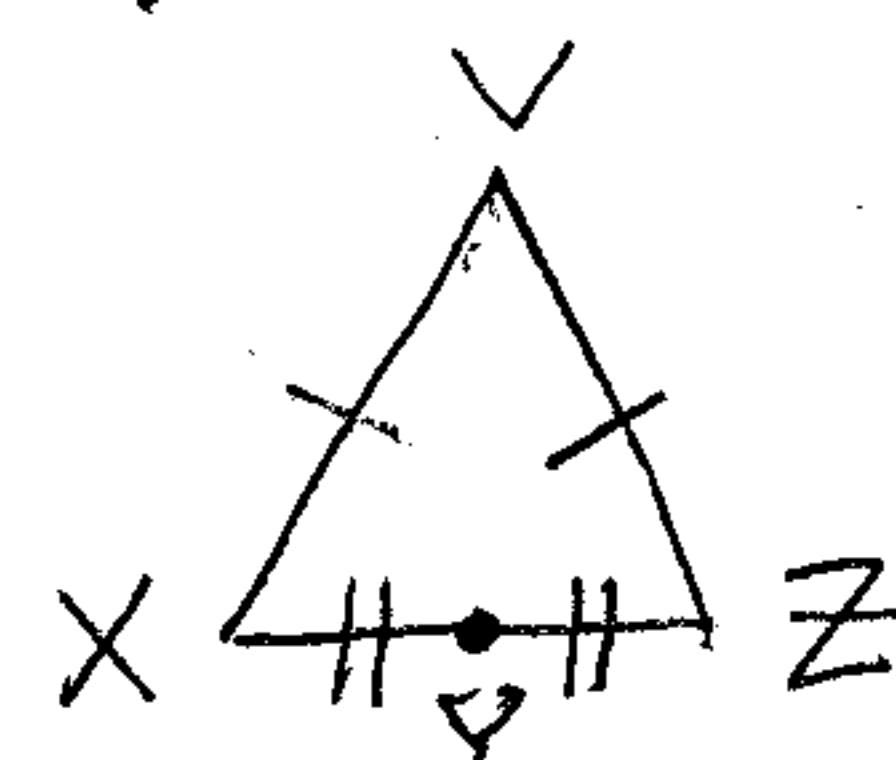
$$y = 15 \Rightarrow \text{perim.} = 6y + 24 = 6(15) + 24 = \textcircled{114}$$

14. A very sneaky problem!

These are the only givens:

Since all the givens of the problem are shown here, points

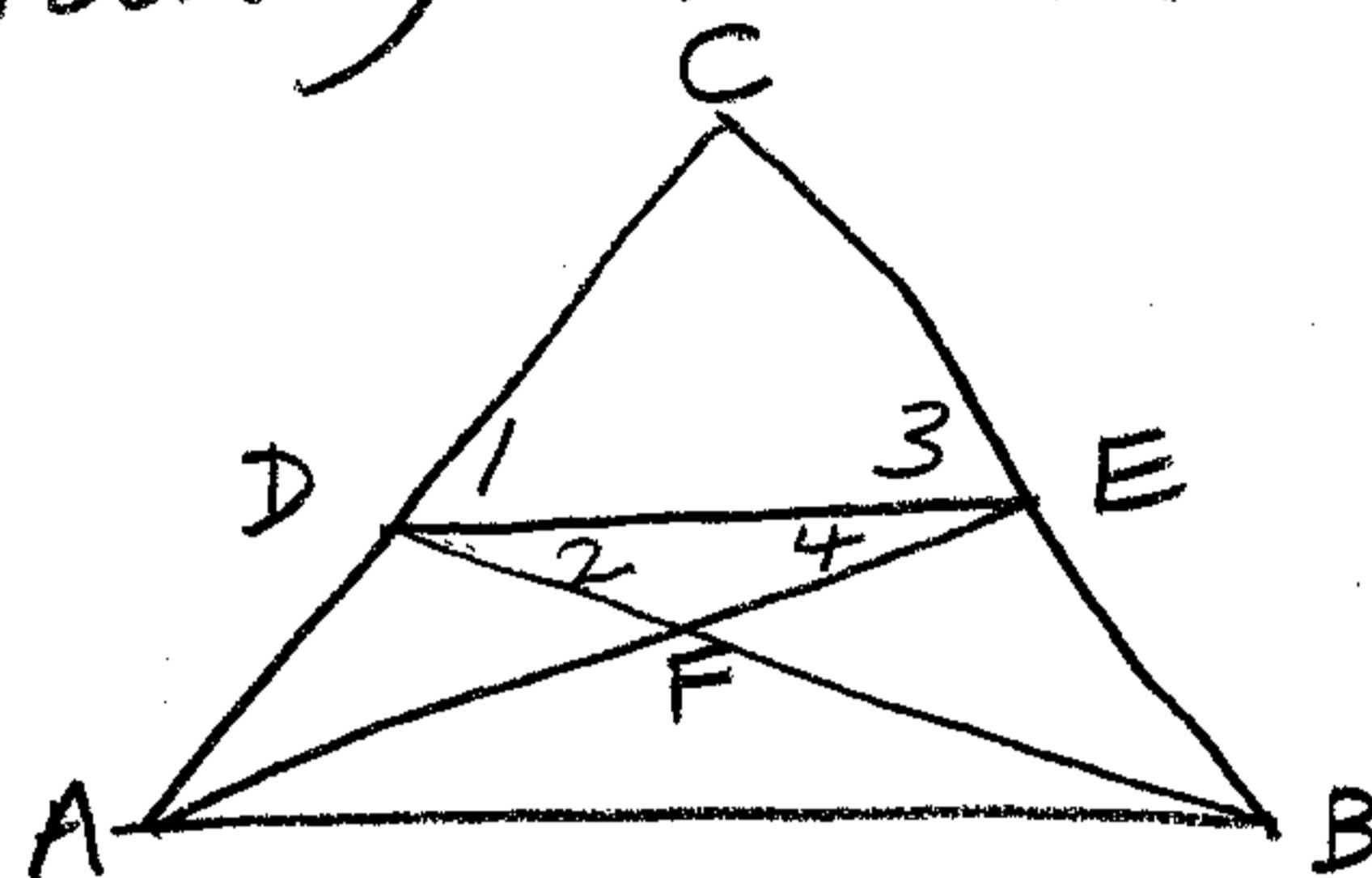
W and A are "free to roam." It cannot be proved that $\overline{WY} \cong \overline{YA}$.



18. This is an excellent problem. Try to work as much of it as possible without looking ahead in the proof.

Given: $\overline{AC} \cong \overline{BC}$
 $\angle 1 \cong \angle 3$

Prove: $\triangle DFE$ is isosceles



First, some forward chaining:

$$\begin{aligned}\angle 1 &\cong \angle 3 \Rightarrow DC = EC \text{ by ITT} \\ \therefore AC &= BC \text{ (given)} \Rightarrow AD = BE \text{ by subtraction}\end{aligned}$$

Now, some backward chaining:

The conclusion would be easy if we could show $\angle 2 \cong \angle 4$ (pre-goal), which in turn would be easy if we could show $\triangle AED \cong \triangle DBE$ (pre-pre-goal).

The hard part is figuring out how to prove $\triangle AED \cong \triangle DBE$. Struggle with this before peeking ahead!

OK, we have to assume that you either peeked (in which case you may have trouble in a testing situation) or struggled and found the resources within yourself to see the necessary path: SAS. If the latter is true, then congratulations! Doesn't it feel good?

You see, $\overline{DE} \cong \overline{DE}$ (refl.), $\angle ADE \cong \angle BED$ (both supp. to $\angle 1$ or $\angle 3$, which are \cong), and $\overline{AD} \cong \overline{BE}$ by the forward chaining from above. We need only write this up:

1. $\overline{AC} \cong \overline{BC}$
2. $\angle 1 \cong \angle 3$
3. $\overline{DC} \cong \overline{EC}$
4. $\overline{AD} \cong \overline{BE}$
5. $\angle ADE$ supp. $\angle 1$,
 $\angle BED$ supp. $\angle 3$
6. $\angle ADE \cong \angle BED$
7. $\overline{DE} \cong \overline{DE}$
8. $\triangle ADE \cong \triangle BDE$
9. $\angle 4 \cong \angle 2$
10. $\triangle DFE$ is isosceles

1. G
2. G
3. ITT ($\triangle \Rightarrow \triangle$)
4. Subtr. Prop. (1, 3)
5. Diag.
6. Supps. of \cong Ls are \cong
7. Refl.
8. SAS (4, 6, 7)
9. CPCTC
10. ITT ($\triangle \Rightarrow \triangle$)

□

Selected problems, pp. 165-167

18. Let x = measure of unknown \angle

$90-x$	" " "	\angle 's comp.
$180-x$	" " "	\angle 's supp.

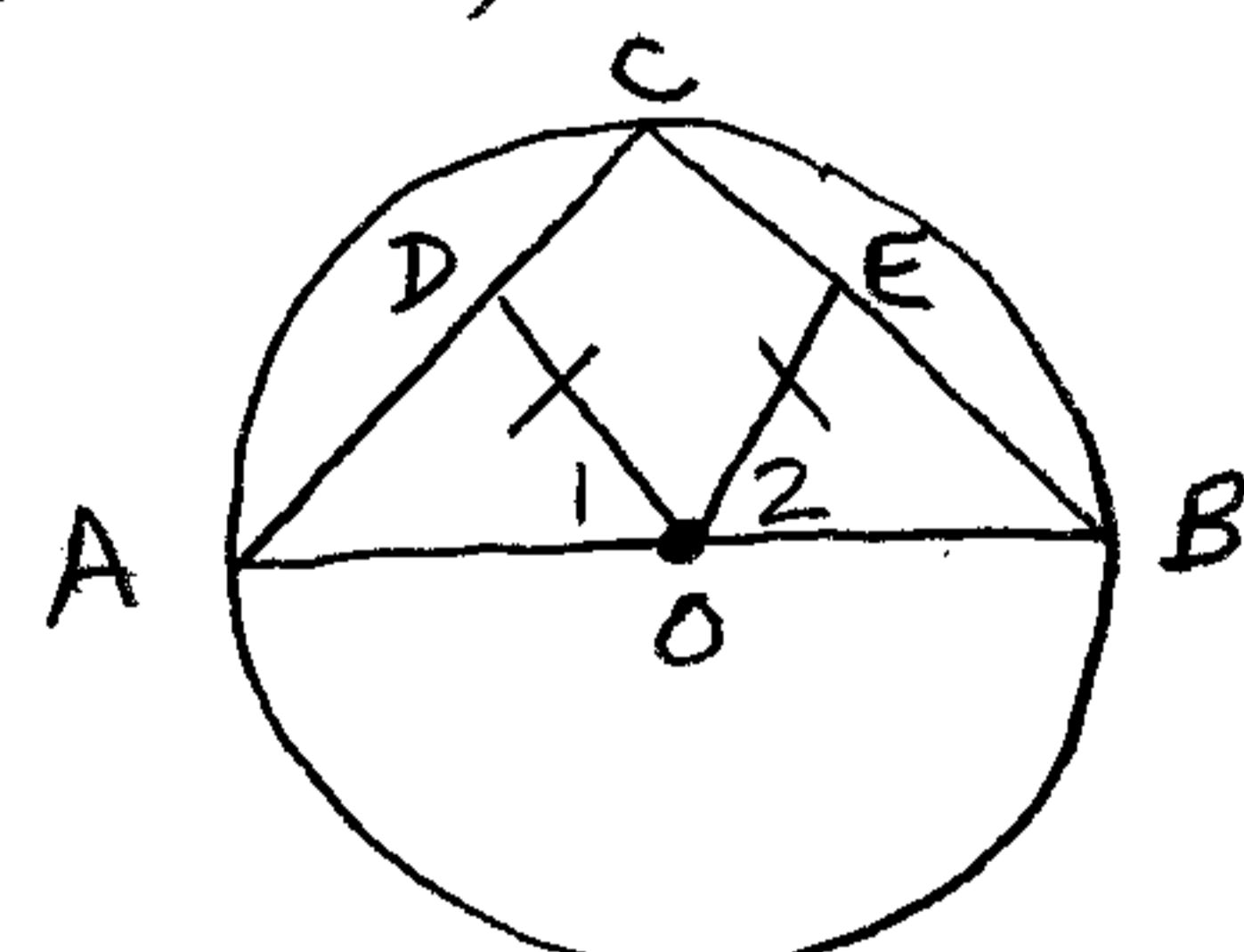
$$180-x = 40 + 2(90-x)$$

$$180-x = 40 + 180 - 2x$$

$$-x = 40 - 2x$$

$$x = 40 \Rightarrow 90-x = 50, \text{ half of which is } \boxed{25}$$

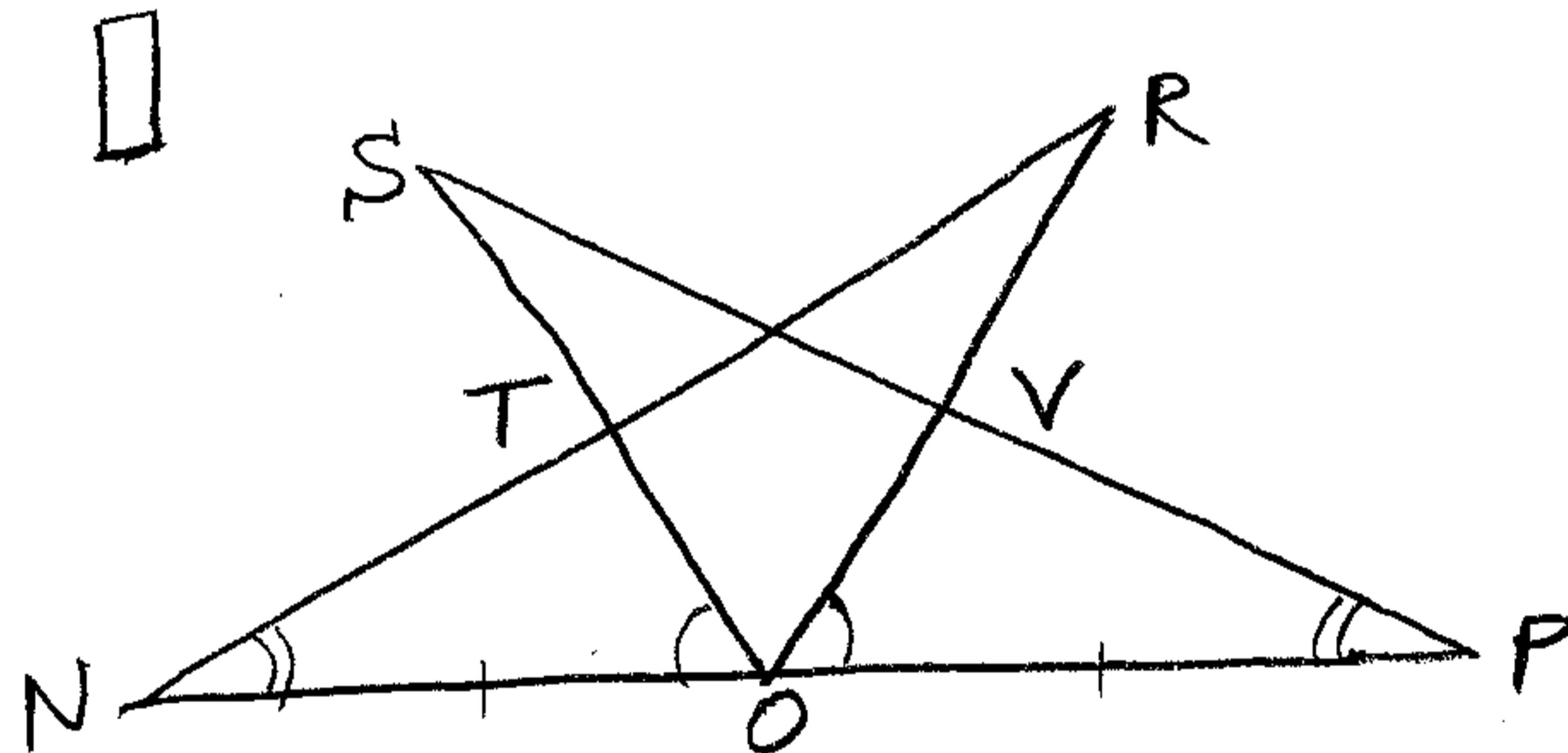
23. Given: $\odot O$
 $\overline{OD} \cong \overline{OE}$
 $\angle DOB \cong \angle COA$
- Prove: $\overline{CD} \cong \overline{CE}$



First, some forward chaining: $\overline{AO} \cong \overline{OB}$ (radii), and $\angle 1 \cong \angle 2$ by subtraction. That means $\triangle DOA \cong \triangle EO B$ by SAS. The rest follows if you use CPCTC twice and apply subtraction. Think it through before you turn the page...

1. $\odot O$
2. $\overline{OA} \cong \overline{OB}$
3. $\angle DOB \cong \angle EOA$
4. $\angle DOE \cong \angle DOB$
5. $\angle 1 \cong \angle 2$
6. $\overline{OD} \cong \overline{OE}$
7. $\triangle DOA \cong \triangle EOB$
8. $\overline{AD} \cong \overline{BE}$
9. $\angle A \cong \angle B$
10. $\overline{AC} \cong \overline{BC}$
11. $\overline{CD} \cong \overline{CE}$

1. G
2. Radii of a \odot are \cong
3. G
4. Refl.]
5. Subtr. Prop. (3, 4)
6. G
7. SAS (2, 5, 6)
8. CPCTC
9. CPCTC
10. ITT ($\Delta \Rightarrow \Delta$)
11. Subtr. Prop. (10, 8)



24. Given: $\angle NOT \cong \angle POV$
 O mdpt. \overline{NP}
 $\angle N \cong \angle P$
 Prove: $\overline{ST} \cong \overline{RV}$

1. $\angle NOT \cong \angle POV$
2. O mdpt. \overline{NP}
3. $\overline{ON} \cong \overline{OP}$
4. $\angle N \cong \angle P$
5. $\triangle NOT \cong \triangle POV$
6. $\overline{OT} \cong \overline{OV}$
7. $\angle SOR \cong \angle SOR$
8. $\angle NOR \cong \angle POS$
9. $\triangle NOR \cong \triangle POS$
10. $\overline{OS} \cong \overline{OR}$
11. $\overline{ST} \cong \overline{RV}$

1. G
2. G
3. Def. mdpt.
4. G
5. ASA (1, 3, 4)
6. CPCTC
7. Refl.]
8. Add. Prop. (1, 7)
9. ASA (4, 3, 8)
10. CPCTC
11. Subtr. Prop. (10, 6)