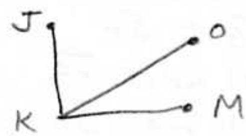
Mr. Hansen
Geom. HW due 9/29/08

Ch. 2 Review Problems #1–38

1. Given: $\overline{JK} \perp \overline{KM}$
   Prove: $\angle JKO$ comp. $\angle OKM$

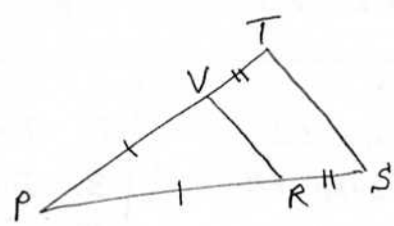


1. $\overline{JK} \perp \overline{KM}$	1. G
[2. $\angle JKM$ is right	[2. Def. $\perp$
3. $m\angle JKM = 90$	3. Def. rt. $\angle$
4. $m\angle JKO + m\angle OKM = m\angle JKM$	4. $\angle$ Addition
5. $m\angle JKO + m\angle OKM = 90$]	5. Subst. (3,4)]
6. $\angle JKO$ comp. $\angle OKM$	6. Def. comp. [measures sum to 90]

$\square$

[Note: By virtue of our class agreement made at the beginning of Chapter 2, steps 2–5 are optional. Throughc these homework solutions, square brackets denote optional work or comments that would not be included in your o HW paper.]

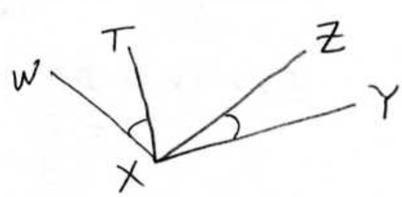
2. Given: $\overline{PV} \cong \overline{PR}$
   $\overline{VT} \cong \overline{RS}$
   Prove: $\overline{PT} \cong \overline{PS}$



1. $\overline{PV} \cong \overline{PR}$	1. G
2. $\overline{VT} \cong \overline{RS}$	2. G
3. $\overline{PT} \cong \overline{PS}$	3. Add. prop.

$\square$

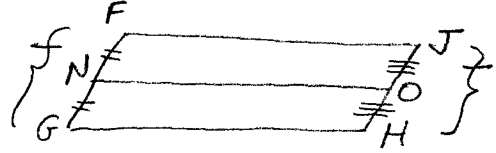
3. Given: $\angle WXT \cong \angle YXZ$
   Prove: $\angle WXZ \cong \angle TXY$



1. $\angle WXT \cong \angle YXZ$	1. G
[2. $\angle TXZ \cong \angle TXZ$	2. Reflexive]
3. $\angle WXZ \cong \angle TXY$	3. Add. prop.

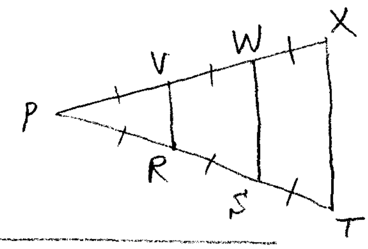
$\square$

4. Given: $\overline{FG} \cong \overline{JH}$
N midpt. $\overline{FG}$
O midpt. $\overline{JH}$
Prove: $\overline{NG} \cong \overline{OH}$



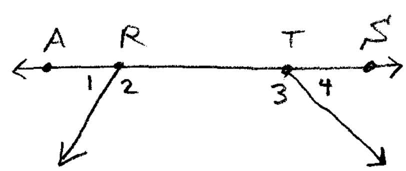
1. $\overline{FG} \cong \overline{JH}$	1. G
2. N mdpt. $\overline{FG}$	2. G
3. O mdpt $\overline{JH}$	3. G
4. $\overline{NG} \cong \overline{OH}$	4. Div. Prop.

5. Given: $\overline{RV}$, $\overline{SW}$ tris. $\overline{PT}$, $\overline{PX}$
$\overline{ST} \cong \overline{WX}$
Prove: $\overline{PT} \cong \overline{PX}$



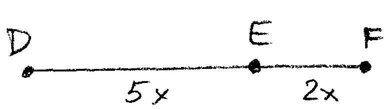
1. $\overline{RV}$, $\overline{SW}$ tris. $\overline{PT}$, $\overline{PX}$	1. G
2. $\overline{ST} \cong \overline{WX}$	2. G
3. $\overline{PT} \cong \overline{PX}$	3. Mult. Prop.

6. Given: Diag.
$\angle 1 \cong \angle 4$
Prove: $\angle 2 \cong \angle 3$



1. Diag.	1. G
2. $\angle 1$ supp. $\angle 2$	2. Diag.
3. $\angle 3$ supp. $\angle 4$	3. Diag.
4. $\angle 1 \cong \angle 4$	4. G
5. $\angle 2 \cong \angle 3$	5. Supps. of $\cong$ $\angle$s are $\cong$

7.

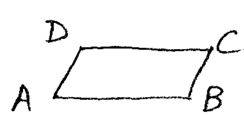


Let $DE = 5x$, $EF = 2x$

$5x + 2x = DF = 21$ cm (given)
$7x = 21$
$x = 3 \Rightarrow EF = 2(3) = 6$ cm

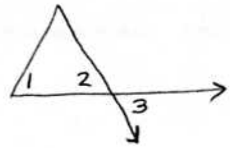
[units required for full credit]

8. Given: $\angle A$ supp. $\angle D$
$\angle A \cong \angle C$
Prove: $\angle C$ supp. $\angle D$



1. $\angle A$ supp. $\angle D$	1. G
2. $\angle A \cong \angle C$	2. G
3. $\angle C$ supp. $\angle D$	3. Subst. (2, 1)

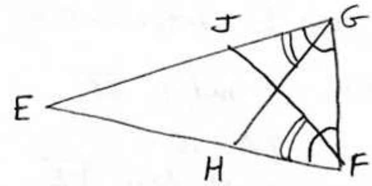
9. Given: $\angle 1 \cong \angle 3$
Prove: $\angle 1 \cong \angle 2$



1. $\angle 1 \cong \angle 3$	1. G
2. $\angle 3 \cong \angle 2$	2. Vert. $\angle$s
3. $\angle 1 \cong \angle 2$	3. Transitive (1,2)

□

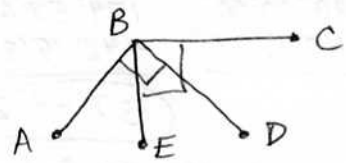
10. Given: $\angle EGF \cong \angle EFG$
$\angle EGH \cong \angle EFJ$
Prove: $\angle HGF \cong \angle JFG$



1. $\angle EGF \cong \angle EFG$	1. G
2. $\angle EGH \cong \angle EFJ$	2. G
3. $\angle HGF \cong \angle JFG$	3. Subtr. Prop.

□

11. Given: $\angle ABD$ is rt.
$\angle CBE$ is rt.
Prove: $\angle ABE \cong \angle CBD$



1. $\angle ABD$ is rt.	1. G
2. $\angle CBE$ is rt.	2. G
3. $\angle ABE \cong \angle CBD$	3. All rt. $\angle$s are $\cong$
4. $\angle EBD \cong \angle EBD$	4. Reflexive
5. $\angle ABE \cong \angle CBD$	5. Subtr. Prop.

□

12. Let x = meas. of one unknown $\angle$
$90-x$ = " " other " " (since comp.)

$x = 6 + 2(90-x)$
$x = 6 + 180 - 2x$
$3x = 186$
$x = 62 \Rightarrow 90 - x = 28$

62 is the larger measure

13. Let x = measure of the unknown $\angle$

$90-x$ = " " " " $\angle$'s comp.

$180-x$ = " " " " $\angle$'s supp.

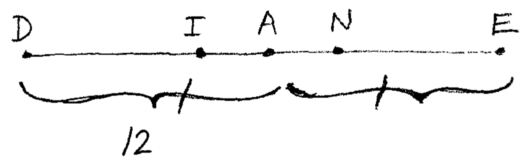
$$180-x=5(90-x)$$
$$180-x=450-5x$$
$$4x=270$$
$$x=67.5 \Rightarrow 90-x=22.5$$

14. Two non $\perp$ intersecting lines are oblique.

15. Given: A mdpt. $\overline{DE}$

$DA=12$

I, N tris. $\overline{DE}$



$DE = 2(12) = 24$

$DI = IN = NE = \frac{1}{3}(24) = 8$

Since $AE = 12$ by def. of mdpt., $AN = AE - NE = 12 - 8 = 4$

16. a) Supp. of $83° = 180° - 83° = 97°$ Comp. of $83° = 90° - 83° = 7°$

b)

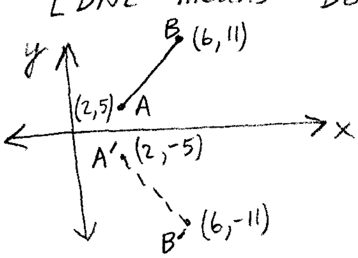
$$\begin{array}{r} 180°00'00'' \\ -\ 42°15'38'' \\ \hline \end{array} \quad \begin{array}{r} 179°59'60'' \\ -\ 42°15'38'' \\ \hline 137°44'22'' \end{array} \qquad \begin{array}{r} 90°00'00'' \\ -42°15'38'' \\ \hline \end{array} \quad \begin{array}{r} 89°59'60'' \\ -42°15'38'' \\ \hline 47°44'22'' \end{array}$$

[Shortcut: Use the fact that an angle's supplement is 90° larger than the complement. This fact is true for all acute angles.]

c) Supp. of $97° = 180° - 97° = 83°$ Comp. of $97°$ = DNE since 97° is no

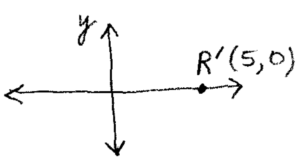
[DNE means "Does Not Exist."]

17.



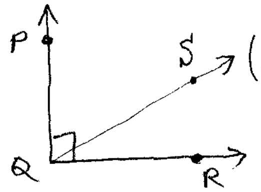
By insp., (2,-5) and (6,-11) are the reflected endpoints.

18.



R was rotated ↷ and then ↶ (net of ↱ or 90° clockwise) to end up a (5,0). ∴ R started at "12 o'clock position" of (0,5)

19.



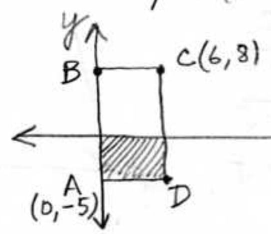
(random ray $\overrightarrow{QS}$)

a) Since $\angle PQS$ is always acute, $\angle SQR$ is always complementary to it. P(comp.) = 1

b) $P(0° < \angle PQS < 45°) = \frac{1}{2}$ by insp.

MH
p.5 (sheet 3)

20. Given: rectangle ABCD



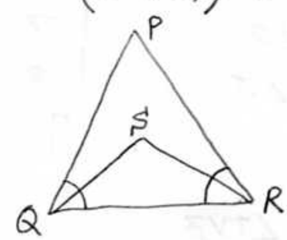
a) B must have same x-coord. as A, same y-coord. as C.
$\therefore B = (0,8)$

D must have same x-coord. as C, same y-coord. as A.
$\therefore D = (6,-5)$

b) $P(\text{shaded region} \mid \text{rect. } ABCD) = \dfrac{\text{shaded area}}{\text{total area}} = \dfrac{6\cdot 5}{l \cdot w}$

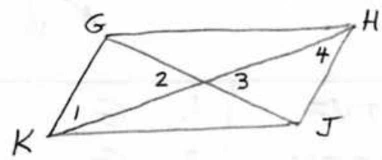
$= \dfrac{30}{(8-(-5))\cdot 6} = \dfrac{30}{13\cdot 6} = \dfrac{5}{13}$

21. Given: $\angle PQR \cong \angle PRQ$
$\overrightarrow{QS}$ bis. $\angle PQR$
$\overrightarrow{RS}$ bis. $\angle PRQ$
$\angle PQR = 87^\circ 26'$



By subst., $\angle PRQ = 87^\circ 26'$.
By def. of bis., $\angle PRS = \frac{1}{2}(\angle PRQ) = \frac{1}{2}(87^\circ 26') = 43\frac{1}{2}^\circ + 13' = 43^\circ 30' +$
$= 43^\circ 43'$

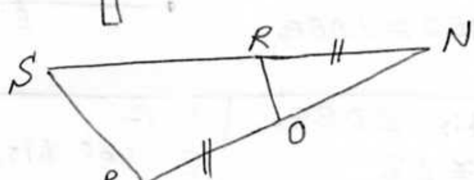
22. Given: $\angle 1$ comp. $\angle 3$
$\angle 4$ comp. $\angle 2$
Prove: $\angle 1 \cong \angle 4$



1. $\angle 1$ comp. $\angle 3$	1. G
2. $\angle 2 \cong \angle 3$	2. Vert. $\angle$s
3. $\angle 1$ comp. $\angle 2$	3. Subst. (2,1)
4. $\angle 4$ comp. $\angle 2$	4. G
5. $\angle 1 \cong \angle 4$	5. $\angle$s comp. to the same $\angle$ are $\cong$

$\square$

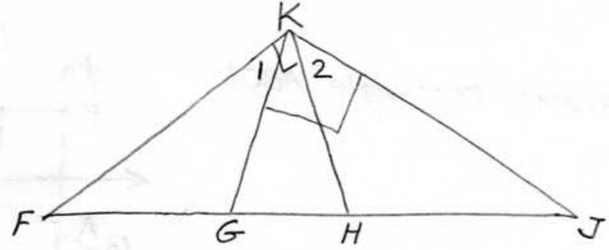
23. Given: O mdpt. $\overline{NP}$
$\overline{RN} \cong \overline{PO}$
Prove: $\overline{RN} \cong \overline{NO}$



1. O mdpt. $\overline{NP}$	1. G
2. $\overline{RN} \cong \overline{PO}$	2. G
3. $\overline{PO} \cong \overline{NO}$	3. Def. mdpt.
4. $\overline{RN} \cong \overline{NO}$	4. Trans. (2,3)

$\square$

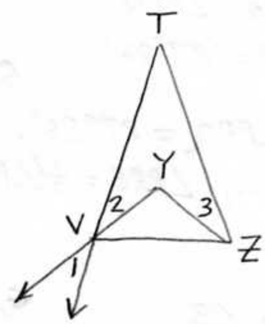
24. Given: $\angle F \cong \angle 1$
$\angle J \cong \angle 2$
$\overline{FK} \perp \overline{KH}$
$\overline{GK} \perp \overline{KJ}$
Prove: $\angle F \cong \angle J$



1. $\overline{FK} \perp \overline{KH}$, $\overline{GK} \perp \overline{KJ}$	1. G
2. $\angle FKJ$, $\angle GKJ$ are rt.	2. Def. $\perp$
3. $\angle FKJ \cong \angle GKJ$	3. All rt. $\angle$s are $\cong$
[4. $\angle GKH \cong \angle GKH$	4. Reflexive]
5. $\angle 1 \cong \angle 2$	5. Subtr. Prop.
6. $\angle F \cong \angle 1$	6. G
7. $\angle J \cong \angle 2$	7. G
8. $\angle F \cong \angle J$	8. Trans. (6, 5, 7)

∎

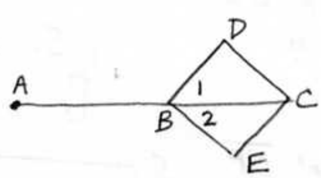
25. Given: $\overrightarrow{VY}$ bis. $\angle TVZ$
$\overrightarrow{ZY}$ bis. $\angle TZV$
$\angle TVZ \cong \angle TZV$
Prove: $\angle 3 \cong \angle 1$



1. $\overrightarrow{VY}$ bis. $\angle TVZ$	1. G
2. $\overrightarrow{ZY}$ bis. $\angle TZV$	2. G
3. $\angle TVZ \cong \angle TZV$	3. G
4. $\angle 3 \cong \angle 2$	4. Div. Prop.
5. $\angle 2 \cong \angle 1$	5. Vert. $\angle$s
6. $\angle 3 \cong \angle 1$	6. Trans. (4, 5)

∎

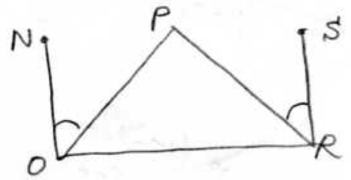
26. Given: $\overrightarrow{BC}$ bis. $\angle DBE$
Prove: $\angle ABD \cong \angle ABE$



1. $\overrightarrow{BC}$ bis. $\angle DBE$	1. G
2. $\angle 1 \cong \angle 2$	2. Def. bis.
3. $\angle ABD$ supp. $\angle 1$	3. Diag.
4. $\angle ABE$ supp. $\angle 2$	4. Diag.
5. $\angle ABD \cong \angle ABE$	5. Supps. of $\cong$ $\angle$s are $\cong$

∎

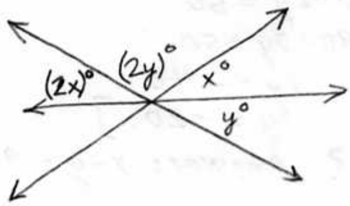
27. Given: $\angle NOP \cong \angle SRP$
$\angle NOP$ comp. $\angle POR$
$\angle SRP$ comp. $\angle PRO$
Prove: $\angle POR \cong \angle PRO$



1. $\angle NOP \cong \angle SRP$	1. G
2. $\angle NOP$ comp. $\angle POR$	2. G
3. $\angle SRP$ comp. $\angle PRO$	3. G
4. $\angle POR \cong \angle PRO$	4. Comps. of $\cong$ $\angle$s are $\cong$

$\square$

28.



By supp. $\angle$s visible in diagram,

$$2x + 2y + x = 180$$
$$3x + 2y = 180$$
$$3x = 180 - 2y$$
$$x = 60 - \tfrac{2}{3}y \quad (*)$$

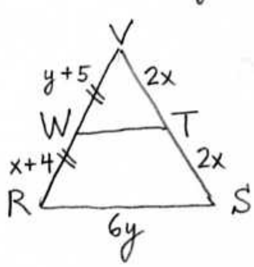
By vert. $\angle$s visible in diag., $y = 2x$. Substitute into the "*" equation to get

$$x = 60 - \tfrac{2}{3}(2x)$$
$$x = 60 - \tfrac{4}{3}x$$
$$\tfrac{7}{3}x = 60$$
$$x = \tfrac{3}{7}(60) = \frac{180}{7} \Rightarrow y = 2x = \frac{360}{7}$$

Check: $2x + 2y + x = 2\left(\frac{180}{7}\right) + 2\left(\frac{360}{7}\right) + \frac{180}{7} = \frac{1260}{7} = 180$ ✓

$2y + x + y = 2\left(\frac{360}{7}\right) + \frac{180}{7} + \frac{360}{7} = \frac{1260}{7} = 180$ ✓

29.



By def. of mdpt., $RW = WV$

$$x + 4 = y + 5$$
$$x = y + 1 \quad (1)$$

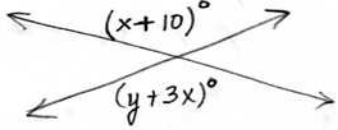
By div. prop., $VS = VR$

$$4x = x + 4 + y + 5$$
$$3x = y + 9$$
$$x = \tfrac{1}{3}y + 3 \quad (2)$$

By trans. (1,2) $y + 1 = \tfrac{1}{3}y + 3$

$$\tfrac{2}{3}y = 2$$
$$y = \tfrac{3}{2}(2) = 3 \Rightarrow x = y + 1 = 4$$

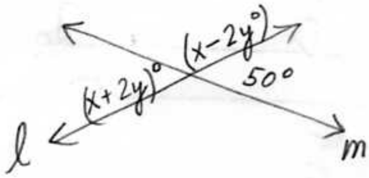
$\therefore$ perim. $= y + 5 + x + 4 + 2x + 2x + 6y$
$= 3 + 5 + 4 + 4 + 2(4) + 2(4) + 6(3) = 50$

30. 

By vert. ∠s, $y+3x = x+10$

$y+2x = 10$

$y = 10-2x$

31. 

Along line ℓ, $(x+2y)+(x-2y)=180$

$2x = 180$

$x = 90$

Along line m, $x-2y+50=180$

$90-2y+50=180$

$-2y = 40$

$y=-20$

[Or, you could use vert. ∠s with $x+2y=50$

$90+2y=50$

$2y=-40$

$y=-20$.]

By how much does x exceed y? Answer: $x-y = 90-(-20) = (11$

Check: ℓ has ∠ measures $x \pm 2y = 90 \pm 2(-20) =$ 50 and 130 ✓

$130°$ is supp. to $50°$ ✓

$50°$ matches vert. ∠ of $50°$ ✓

32. Let x = measure of unknown ∠

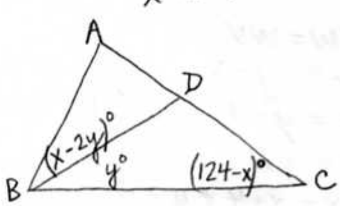
$90-x =$ " " " ∠'s comp.

$180-x =$ " " " ∠'s supp.

$180-x = 20+2(90-x)$

$180-x = 20+180-2x$

$x = 20 \Rightarrow$ half the comp. $= \frac{1}{2}(90-x) = \frac{1}{2}(90-20) = 35$

33. 

Given: $\overrightarrow{BD}$ bis. ∠ABC

a) By def. bis., $x-2y=y$

$x=3y$

b) If $\angle DBC \cong \angle C$, then $y=124-x$.

c) $y=124-3y$

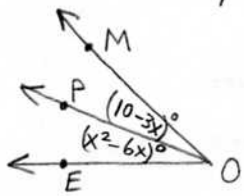
$4y=124$

$y=31 \Rightarrow x=3y=3(31)=93$

$\therefore m\angle C = 124-x = 124-93 = 31$

MH
p. 9 (sheet 5)

34. Given: $\overrightarrow{OP}$ bis. $\angle MOE$

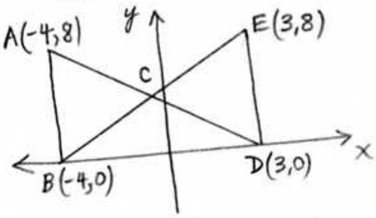


$x^2 - 6x = 10 - 3x$
$x^2 - 3x - 10 = 0$
$(x+2)(x-5) = 0$
$x = -2$ or $\underbrace{x = 5}$

must reject since $m\angle MOP$ would be $10-3(5) = -5$ $(\rightarrow\leftarrow)$

$\therefore m\angle MOP = 10 - 3x = 10 - 3(-2) = 16$
$m\angle MOE = 2m\angle MOP = 2(16) = 32$

35.



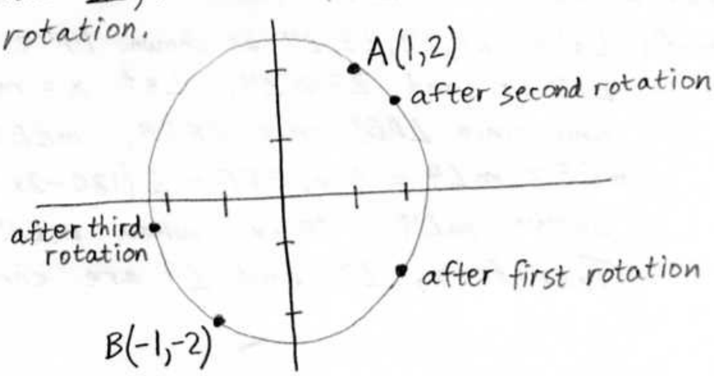
a) $A_{\Delta BDE} = \frac{1}{2}bh$
$= \frac{1}{2}(7)(8) =$ 28 sq. units

b) $A_{\Delta ABC} = A_{\Delta EDC}$ by insp.
Same area since E and D are the reflections of A and B across the line $x = -1$. Thus ΔABC and ΔEDC are mirror images of each other. [Note: We have not yet proved that mirror-image triangles are congruent, but the book apparently wants us to make that reasonable leap of faith here.]

36. Start at $(1,2)$. Rotate 100° cw, then 80° ccw, then 210° cw, and finally 50° ccw. This is a net of 180° cw, i.e., reflection across the origin.

a) $B = (-1,-2)$

b) A is in Quadrant I. Rotation 100° cw goes to Quadrant IV, but 80° ccw returns us to Quadrant I. The third rotation takes t point to Quadrant III, where it remains. Answer: Quadrant I af the second rotation.



37. Here are all possible times of cuckoo:

Time	Angle	Acute?
12:00	0°	
12:30	165°	
1:00	30°	YES
1:30	135°	
2:00	60°	YES
2:30	105°	
3:00	90°	
3:30	75°	YES
4:00	120°	
4:30	45°	YES
5:00	150°	
5:30	15°	YES
6:00	180°	
6:30	15°	YES
7:00	150°	
7:30	45°	YES
8:00	120°	
8:30	75°	YES
9:00	90°	
9:30	105°	
10:00	60°	YES
10:30	135°	
11:00	30°	YES
11:30	165°	

10 acute angles out of 24 total possibilities

Bippy is making a bet on an outcome that is less likely than a 50/50 proposition. The probability for Bippy to win is $\frac{10}{24}$, meaning that the odds against Bippy are 14 to 10. Tippy should wager $14 against Bippy's $10 to make the bet fair.

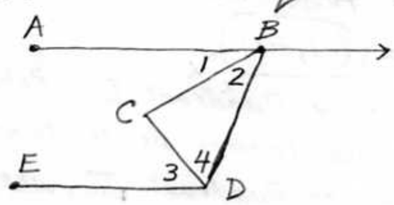
But note: This is a very silly problem, since presumably Bippy (who has not been sleeping) would know what time it is. Therefore, the laws of odds and probability do not apply. Bippy would offer the bet only if he knew he would win the bet, in which case the real answ for the amount of money Tippy should be required to put up is $0.

Given: $\angle ABD$ supp. $\angle EDB$

$\overrightarrow{BC}$ bis. $\angle ABD$

$\overrightarrow{DC}$ bis. $\angle BDE$

Prove: $\angle CBD$ comp. $\angle BDC$



¶ proof: Label $\angle 1, \angle 2, \angle 3, \angle 4$ as shown. Note that by def. of bis., $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. Let $x = m\angle 1 = m\angle 2$. Then $m\angle ABD = 2x$, and since $\angle ABD$ supp. $\angle EDB$, $m\angle EDB = 180 - 2x$. By def. of bi $m\angle 3 = m\angle 4 = \frac{1}{2} m\angle EDB = \frac{1}{2}(180 - 2x) = 90 - x$. We have shown $m\angle 4 = 90 - x$ when $m\angle 2$ was defined as x. Therefore, $\angle 2$ and $\angle 4$ are complementary. $\square$