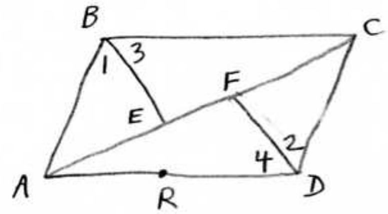
Mr. Hansen
Geom. Review Problems
pp. 54–59 (9/15/08)

1.



a) Line containing A, R, and D = $\overleftrightarrow{AR}, \overleftrightarrow{RA}, \overleftrightarrow{AD}, \overleftrightarrow{DA}, \overleftrightarrow{RD}, \overleftrightarrow{DR}$.

b) $\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC}$

c) $\angle 2 \cap \angle 4 = \overrightarrow{DF}$ [note: $\overline{DF}$ is technically incorrect]

d) horiz. ray w/ endpt. C = $\overrightarrow{CB}$

e) $\angle BAD \approx 60°$ as drawn
$\angle 2 \approx 60°$ as drawn
$\angle ABC \approx 120°$ as drawn

f) $\angle FCE = \angle DCE$ (not different)

g) $\angle B$ is ambiguous

h) $\overrightarrow{EC} \cup \overrightarrow{FA} = \overleftrightarrow{AC}$

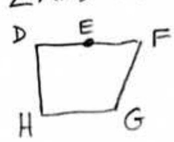
i) $\overrightarrow{EC} \cap \overrightarrow{FA} = \overline{EF}$

j) $\overrightarrow{BA} \cup \overrightarrow{BE} = \angle 1$

k) $\overleftrightarrow{AC} \cap \overleftrightarrow{DR} = A$

l) $\angle AFD \cap \overline{CE} = \overline{EF}$

2.



a) $\angle H$ appears rt. (but cannot be assumed)

b) $\angle G$ appears obtuse (" " " ")

c) $\angle GFE$ appears acute (" " " ")

d) $\angle DEF$ <u>is</u> straight (this is the <u>only</u> one that can be assumed!)

e) $\angle HDF$ appears rt. (but cannot be assumed)

3. a)
$$\begin{array}{r} 43°15'17'' \\ +\ 25°49'18'' \\ \hline 68°64'35'' \end{array} = 69°04'35''$$

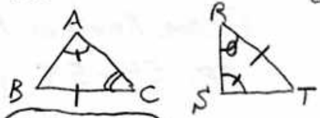
b)
$$\begin{array}{r} 90°00'00'' \\ -\ 39°00'17'' \\ \hline \end{array} \quad \begin{array}{r} 89°60'00'' \\ -39°00'17'' \\ \hline \end{array} \quad \begin{array}{r} 89°59'60'' \\ -39°00'17'' \\ \hline 50°59'43'' \end{array}$$

. a) $46\frac{7}{8}° = 46° + \frac{7}{8}° = 46° + \frac{7}{8}(60') = 46° + \frac{420}{8}' = 46° + 52\frac{1}{2}' = 46°52'30''$

b) $132°06' = 132° + \frac{6}{60}° = 132° + \frac{1}{10}° = 132.1°$

M
p.

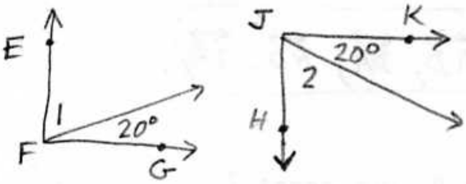
5.



a) $\overline{BC} \cong \overline{RT}$

b) $\angle A \cong \angle S$

6.



a) Given: $m\angle EFG > 90$
$m\angle HJK = 90$

We know $m\angle 2 = 70$ by insp.
However, $m\angle 1$ must be >70 in order to make $\angle EFG$ obtuse.
Thus $\angle 1 \not\cong \angle 2$.

b) Given: $\angle EFG \cong \angle HJK$

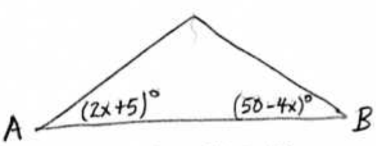
Since $m\angle EFG = m\angle HJK$, subst. gives

$m\angle 1 + 20 = m\angle 2 + 20$

$\therefore\ m\angle 1 = m\angle 2$

$\angle 1 \cong \angle 2$

7.



Given: $\angle A \cong \angle B$

$2x + 5 = 50 - 4x$

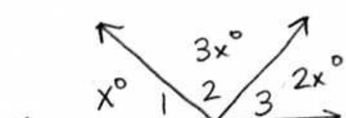
$6x = 45$

$x = \frac{45}{6} = 7.5$

$m\angle A = 2x + 5 = 2(7.5) + 5 = 20$

Check: $m\angle B = 50 - 4x = 50 - 4(7.5) = 20$ ✓

8.



$m\angle 1 + m\angle 2 + m\angle 3 = 180$

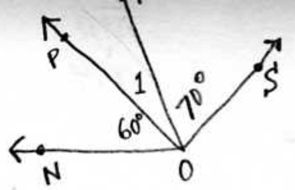
$x + 3x + 2x = 180$

$6x = 180$

$x = 30$

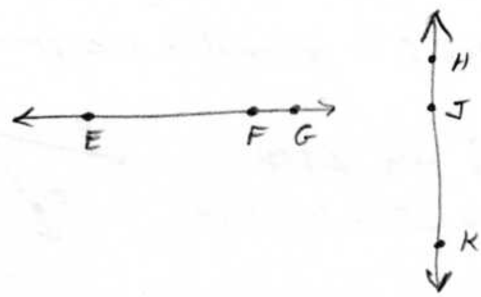
$\therefore \angle 1 = x° = 30°$, $\angle 2 = 3x° = 90°$,

$\angle 3 = 2x° = 60°$

9. 

Assume (bwoc) that $m\angle NOR = m\angle POS = 90$.
If $\angle NOR = 90°$, $\angle 1$ must be $30°$.
But if $\angle POS = 90°$, $\angle 1$ must be $20°$. $(\rightarrow\leftarrow)$
Conclusion: No, $\angle NOR$ and $\angle POS$ cannot both be rt. $\angle$s.

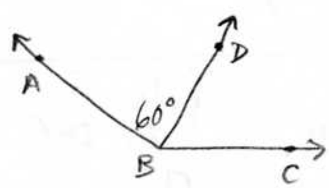
10. Given: Diag.
Prove: $\angle EFG \cong \angle HJK$



1. Diag.	1. G
2. $\angle EFG$ is straight	2. Diag.
3. $\angle HJK$ " "	3. Diag.
4. $\angle EFG \cong \angle HJK$	4. All straight $\angle$s are $\cong$.

∎

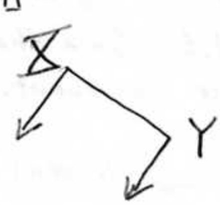
11. Given: $\angle ABC = 130°$
$\angle ABD = 60°$
Prove: $\angle DBC$ is acute



1. $\angle ABC = 130°$	1. G
2. $\angle ABD = 60°$	2. G
3. $\angle DBC = 70°$	3. $\angle$ subtraction
4. $\angle DBC$ is acute	4. Def. acute

∎

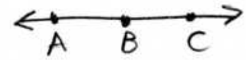
12. Given: $\angle X$ is a rt. $\angle$
$\angle Y$ is a rt. $\angle$
Prove: $\angle X \cong \angle Y$



1. $\angle X$ is rt.	1. G
2. $\angle Y$ is rt.	2. G
3. $\angle X \cong \angle Y$	3. All rt. $\angle$s are $\cong$

∎

13. Given: $\overline{AB} \cong \overline{BC}$
Prove: B is mdpt. of $\overline{AC}$

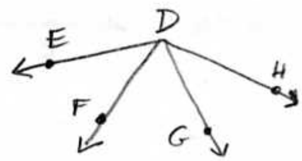


1. $\overline{AB} \cong \overline{BC}$	1. G
[2. B is the bis. pt. of $\overline{AC}$	2. Def. bis. ($2 \cong$ segs.)]
3. B is the mdpt. of $\overline{AC}$	3. Def. mdpt.

$\square$

[Note: Step 2 is optional in the proof above.]

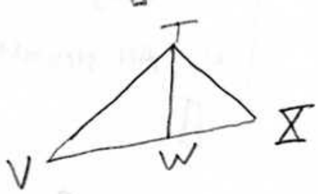
4. Given: $\overrightarrow{DF}$, $\overrightarrow{DG}$ tris. $\angle EDH$
Prove: $\angle EDF \cong \angle FDG \cong \angle DGH$



1. $\overrightarrow{DF}$, $\overrightarrow{DG}$ tris. $\angle EDH$	1. G
2. $\angle EDF \cong \angle FDG \cong \angle DGH$	2. Def. tris.

$\square$

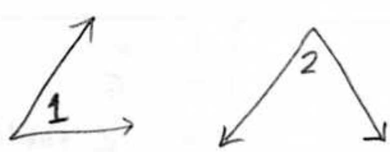
5. Given: $\overrightarrow{TW}$ bis. $\angle VTX$
Prove: $\angle VTW \cong \angle XTW$



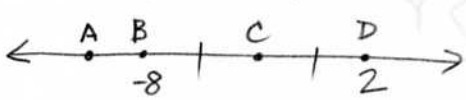
1. $\overrightarrow{TW}$ bis. $\angle VTX$	1. G
2. $\angle VTW \cong \angle XTW$	2. Def. bis.

$\square$

, Given: $\angle 1 = 61.6°$
$\angle 2 = 61\frac{3}{5}°$
Prove: $\angle 1 \cong \angle 2$



¶ proof: Since $\angle 2$ is $61°$ plus $\frac{3}{5}$ of another degree, and since $\frac{3}{5}$ of a degree is $\frac{6}{10}$ or .6 of a degree, $m\angle 2 = 61.6$. Since that matches $m\angle 1$, the angles are congruent. $\square$

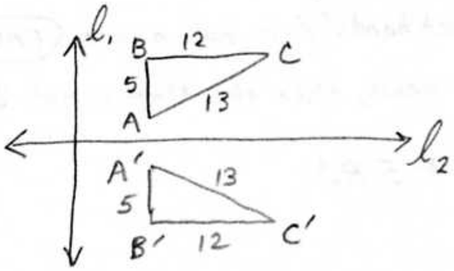


Given: C is mdpt. of $\overline{BD}$

a) Since $BD = 2 - (-8) = 10$, $\frac{BD}{2} = 5$. Add 5 to coord. of B (or subtract 5 from coord. of D) to get C at -3.

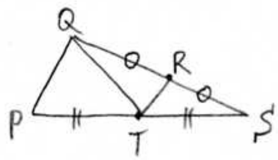
b) If $AD = 15$, D is 15 units to the right of A. By insp., A is at -13.

18.



$A'B' = AB = 5$

19.

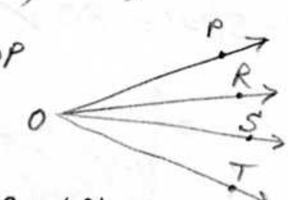


midpoints: R, T
non-mdpts.: Q, S, P

a) $P(\text{mdpt.}) = P(R \text{ or } T) = \frac{2}{5}$

b) $P(2 \text{ mdpts.}) = P(\{R, T\}) = \frac{1}{10}$ since there are 10 ways to select 2 points without regard to order: QR, QS, QT, QP, RS, RT, RP, ST, SP, TP.

20. Given: $\overrightarrow{OR}$, $\overrightarrow{OS}$ tris. $\angle TOP$
$\angle TOP = 40.2°$



$$m\angle POR = \tfrac{1}{3}(40.2°) = \tfrac{1}{3}(40° + \tfrac{1}{5}°)$$
$$= \tfrac{1}{3}(40°) + \tfrac{1}{15}° = \tfrac{1}{3}(39° + 60') + \tfrac{1}{15}(60')$$
$$= 13° + 20' + 4' = 13°24'$$

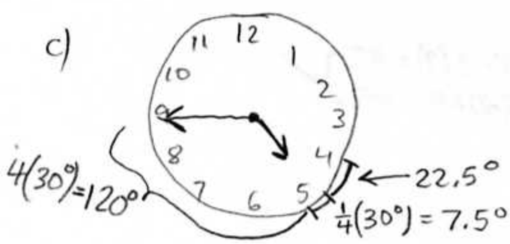
21. a)

$30°$ by insp. since $\frac{1}{12}$ of $360°$

b)

$120° + 20° = 140°$ at 11:20

c)



$120° + 7.5° = 127°30'$

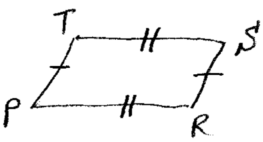
22. "If the time is 2:00, then the ∠ formed by clock hands is acute." TRUE

CONVERSE: If the ∠ formed by clock hands is acute, then the time is 2:00. FALSE

INVERSE: If the time is not 2:00, then the clock hands' ∠ is not acute. FALSE

CONTRAPOSITIVE: If the clock hands' ∠ is not acute, then the time is not 2:00. TRUE

23.



Given: perim. $= 10 + 5RS$

$PR = 26$

Find: RS

$RS = TP$

$PR = TS = 26$

perim. $= PR + RS + ST + TP = 10 + 5RS$

$26 + RS + 26 + RS = 10 + 5RS$

$52 + 2RS = 10 + 5RS$

$42 = 3RS$

$RS = \frac{42}{3} = 14$

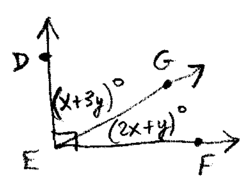
Check: perim. $= 14 + 14 + 26 + 26 = 80$

$10 + 5RS = 10 + 5(14) = 80$ ✓

24. Given: $m\angle DEG = x + 3y$

$m\angle GEF = 2x + y$

$m\angle DEF = 90$



a) $(x+3y) + (2x+y) = 90$

$3x + 4y = 90$

$4y = 90 - 3x$

$y = \frac{90-3x}{4}$

b) If $\angle DEG \cong \angle GEF$, then $x + 3y = 2x + y$

$2y = x$

$y = \frac{x}{2}$

$\frac{90-3x}{4} = \frac{x}{2}$

Cross-mult. to get $4x = 2(90-3x)$

$4x = 180 - 6x$

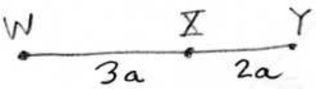
$10x = 180$

$x = 18$

$y = \frac{x}{2} = \frac{18}{2} = 9$ Check: $m\angle DEG = x + 3y = 18 + 3(9) = 45$

$m\angle GEF = 2x + y = 2(18) + 9 = 45$ ✓

M
P.

25. 

Given: $WY = 25$
$WX : XY = 3 : 2$

$$3a + 2a = 25$$
$$5a = 25$$
$$a = 5$$
$$WX = 3a = 15$$

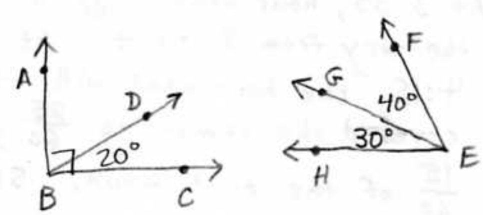
26. Let $a = m\angle A$
$b = m\angle B$

Given: $a = 6 + 2b$
$a + b = 42$

subtract to get $-b = -36 + 2b$
$$-3b = -36$$
$$b = 12$$
$$a = 6 + 2b = 6 + 2(12) = 30$$

Check: $a + b = 30 + 12 = 42$ ✓
Is "a" 6 more than $2b$?
6 more than $2b = 6 + 2(12)$
$= 30$
$a = 30$ ✓

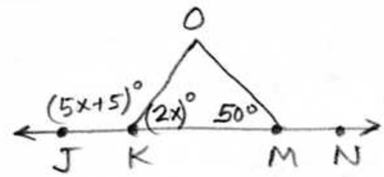
27. Given: $\angle ABC$ is rt.
$\angle DBC = 20°$
$\angle FEG = 40°$
$\angle GEH = 30°$
Prove: $\angle ABD \cong \angle FEH$



1. $\angle ABC$ is rt.	1. G
2. $\angle ABC = 90°$	2. Def. rt. $\angle$
3. $\angle DBC = 20°$	3. G
4. $\angle ABD = 70°$	4. $\angle$ subtraction (2,3)
5. $\angle FEG = 40°$	5. G
6. $\angle GEH = 30°$	6. G
7. $\angle FEH = 70°$	7. $\angle$ addition (5,6)
8. $\angle ABD \cong \angle FEH$	8. Def. $\cong$ (4,7)

$\square$

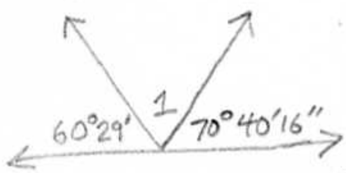
28. Given: $\angle OMK = 50°$
$\angle OKM = (2x)°$
$\angle OKJ = (5x+5)°$



Prove: $\angle OKJ \cong \angle OMN$

¶ proof: Since $\angle OMK = 50°$ and $\angle KMN$ is straight, $\angle OMN = 130°$. Since $\angle JKM$ is straight, $5x+5+2x=180$, from which we obtain $7x = 175$, or $x = 25$. Thus $\angle OKJ = (5x+5)° = (5(25)+5)° = 130°$, which matches $\angle OMN$. □

29.



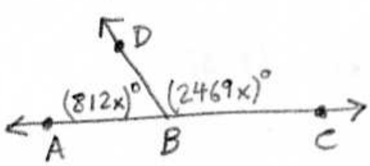
$\angle 1 + 60°29' + 70°40'16" = 180°00'00"$
$\angle 1 + 130°69'16" = 179°60'00"$
$\angle 1 + 131°09'16" = 179°59'60"$

$$\begin{array}{r} 179°59'60" \\ -\ 131°09'16" \\ \hline 48°50'44" \end{array} = \angle 1$$

30.

At 3:35, hour hand is $\frac{35}{60}$ of the way from 3 to 4. At 4:15, the hour hand will have covered the remaining $\frac{25}{60}$, plus $\frac{15}{60}$ of the next hour. Since $\frac{25}{60} + \frac{15}{60} = \frac{40}{60} = \frac{2}{3}$, the total travel is $\frac{2}{3}$ of an hour, and each hour represents 30° as we well know. Final answer: $\frac{2}{3}(30°) = 20°$.

31.

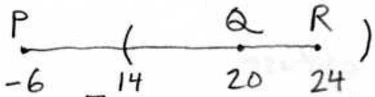


$812x + 2469x = 180$
$3281x = 180$
$x = \frac{180}{3281}$

a) $\angle ABD = (812x)° = 812\left(\frac{180}{3281}\right)° \approx 44.54739° \approx 44.5°$

b) Since $.54739° = .54739\,(60') \approx 32.8436' \approx 33'$,
$\angle ABD \approx 44°33'$

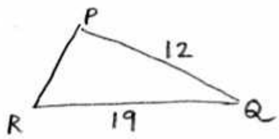
[Calculator not allowed on test. You would have to leave answer for (a) as $\left(\frac{812 \cdot 180}{3281}\right)°$. For part (b), you would simply describe how to multiply the fractional part by 60 to get minutes.]

32. 

Let Z = random point
z = coordinate of random pt. Z

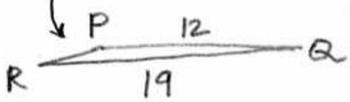
$P(14 < z < 26 \text{ and } Z \in \overline{PR}) = \frac{10}{PR}$

$= \frac{10}{24-(-6)} = \frac{10}{30} = \frac{1}{3}$

33. 

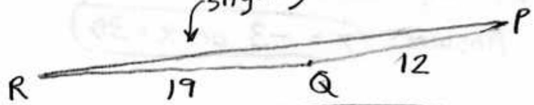
Extreme case #1 ($\angle Q$ very, very small):

slightly >7, since PR=7 only if R,P,Q are collinear

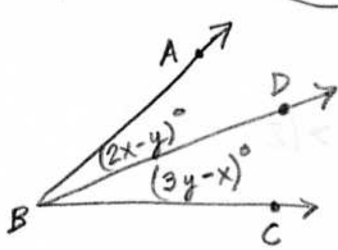


Extreme case #2 ($\angle Q$ very, very large):

slightly <31, since PR=31 only if R,Q,P are collinear



Answer: $7 < PR < 31$

34. 

Given: $\overrightarrow{BD}$ bis. $\angle ABC$
$m\angle ABC = 25$

Find: x and y

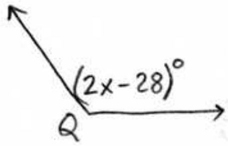
We know $\underbrace{(2x-y)+(3y-x)=25}$, and $\underbrace{2x-y=3y-x}$

$x + 2y = 25$
$x = 25 - 2y$

$3x = 4y$
$3(25-2y) = 4y$
$75 - 6y = 4y$
$75 = 10y$
$y = 7.5$

$x = 25 - 2(7.5)$
$x = 25 - 15$
$x = 10$

Check: $m\angle ABD = 2x-y = 2(10)-7.5 = 12.5$
$m\angle DBC = 3y-x = 3(7.5)-10 = 12.5$ ✓

35. 

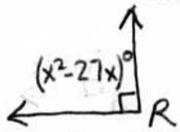
Given: $\angle Q$ is obtuse

a) $m\angle Q > 90$ and $m\angle Q < 180$

b) $90 < 2x - 28 < 180$

$118 < 2x < 208$

$59 < x < 104$

36. 

Given: $\angle R$ is rt.

$x^2 - 27x = 90$

$x^2 - 27x - 90 = 0$

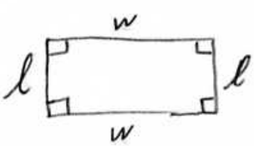
$(x+3)(x-30) = 0$

$x = -3$ or $x = 30$

Check: If $x=-3$, $x^2-27x = (-3)^2 - 27(-3) = 9+81 = 90$ ✓

If $x = 30$, $x^2 - 27x = x(x-27) = 30(30-27) = 90$ ✓

We must keep both roots. Answer: $x = -3$ or $x = 30$

37. 

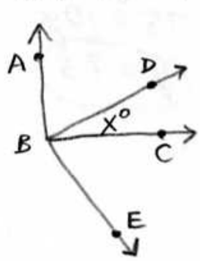
Given: perim. $= 2l + 2w = 20$
$l < 4$

By the first given, $2w = 20 - 2l$.
Since $2l$ is a number less than 8, $20 - 2l > 12$.
We have $2w = 20 - 2l > 12$

$2w > 12$

$w > 6$

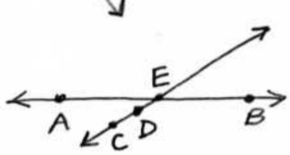
However, there is more! If w were 10, we would no longer have a rectangle since $2w$ would make the full perimeter. Therefore, $6 < w < 10$.

38. 

Given: $\angle ABC$ is rt.
$\angle DBE$ is rt.

Prove: $\angle ABD \cong \angle CBE$

A proof: Let $m\angle DBC = x$ as shown. By $\angle$ subtraction, $m\angle ABD = 90 - x$, and $m\angle CBE = 90 - x$. Since $m\angle ABD = m\angle CBE$, $\angle ABD \cong \angle CBE$. ▯

39. 

By placing C and D on the same side of $\overleftrightarrow{AB}$, we could have $\angle AEC \not\cong \angle DEB$. [Other solutions possible.]