Superpod 3

Work for problem 1(a)

$\left(\frac{dx}{dt}\right)' = 12 - 6t$ $\quad\left(\frac{dy}{dt}\right)' = \frac{1}{1+(t-4)^4} \cdot 4(t-4)^3$

at $t = 2$, $a(t) = \langle 0, -\frac{32}{17} \rangle$

Speed $= \|v(t)\|$

$= \sqrt{12^2 + \ln(17)^2}$

$\boxed{\text{Speed} = \sqrt{144 + (\ln 17)^2}}$

Work for problem 1(b)

$y'(t) = \ln(1 + (t-4)^4) \quad y(0) = 5$

$\int_0^2 y'(t)\,dt + 5 = y(2)$

$y(2) = \int_0^2 \ln(1 + (t-4)^4)\,dt + 5$

$\boxed{y(2) = 13.671\ldots}$

The y-coordinate at point P is 13.671...

Superpod 3

Work for problem 1(c)

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \qquad y'(2) = \ln 17 \qquad \frac{dy}{dx} \text{ where } t=2 = \frac{\ln 17}{12}$$

$$x'(2) = 12$$

$$y - 13.671\ldots = \frac{\ln 17}{12}(x-3)$$

$$\boxed{y = \frac{\ln 17}{12}(x) - \frac{\ln 17}{4} + 13.671\ldots}$$

Work for problem 1(d)

$x'(t)=0$ and $y'(t)=0$

$x'(4) = 48-48=0$

$y'(4) = \ln(1+0) = 0$

at $t=4$, $x'(t)=0$ and $y'(t)=0$. Therefore, the velocity of the object is zero and the object is at rest.

Work for problem 2(a)

$W(15) - R(15) = -121.090\ldots$

No it is not increasing, because $-121.090 < 0$

Work for problem 2(b)

$\int_0^{18} (W(t) - R(t))\,dt = 109.788\ldots$

$+1200$

1309.788

1310 gallons

Superpod 3

Work for problem 2(c)

Minimum occurs when $W(t) - R(t) = 0$

$t = 6.4948\ldots$ and $t = 12.9748\ldots$

gallons $= \int_0^{6.494\ldots} (W(t) - R(t))\,dt + 1200$ → gallons: $\int_0^{12.974} (W(t) - R(t))\,dt + 1200 = 1697.44$

$= 525.242$

Check endpoints:

$t = 0$, gallons $= 1200$

$t = 18$, gallons $= 1309.78$

$t = 6.4947\ldots$ hours

Work for problem 2(d)

when $t = 18$, # of gallons ≈ 1309.78

$\int_{18}^{k} R(t)\,dt = 0 - 1309.78$

$\therefore \int_{18}^{k} R(t)\,dt = 1309.78$

Superpod 3

Work for problem 3(a)

Maximum. Because after $u=2$, the derivative are all either less than zero (when $n=2$, ~~the~~ $f(x)$ is decreasing first) or slightly large than zero but far smaller than 1, So $f(x)$ generally decrease after $n=2$. So $f(x)$ has its maximum at zero.

Work for problem 3(b)

$$\frac{f^{(n)}(x)}{n!}\cdot(x-a)^n$$

$f'(x)=0$ (horizontal tangent line)

$$f''(0)=\frac{(-1)^3(2+1)!}{5^2(1)^2}=\frac{-1\cdot 6}{25}=-\frac{6}{25}$$

2nd degree: $\frac{-\frac{6}{25}}{2\times 1}\cdot(x)^2=-\frac{3}{25}x^2$

3rd degree

$$f'''(0)=\frac{(-1)^4\cdot(3+1)!}{5^3\cdot 4}=\frac{1\cdot \cancel{4}\times 3\times 2\times 1}{125\cdot\cancel{4}}=\frac{6}{125}$$

$$\frac{\frac{6}{125}}{3\times 2\times 1}\cdot x^3=\frac{1}{125}x^3$$

Thus the series at 3rd degree: $6+0-\frac{3}{25}x^2+\frac{1}{125}x^3$

$$\boxed{=6-\frac{3}{25}x^2+\frac{1}{125}x^3}$$

Superpod 3

Work for problem 3(c)

$$L = \lim_{n\to\infty} \left| \frac{t_{n+1}}{t_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+2}(n+2)!}{5^{n+1}\cdot(n)^2}}{(n+1)!} x^{n+1} \cdot \frac{n!}{\frac{(-1)^{n+1}(n+1)!}{5^n \cdot (n-1)^2} \cdot x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-1\cdot n+2}{5\cdot n+1} \cdot \frac{(n-1)^2}{n^2} \cdot x \right|$$

$$= \left| -\frac{1}{5}x \right| \cdot \lim_{n\to\infty} \left| \frac{n+2}{n+1} \right| \cdot \lim_{n\to\infty} \left| \frac{(n-1)^2}{n^2} \right|$$

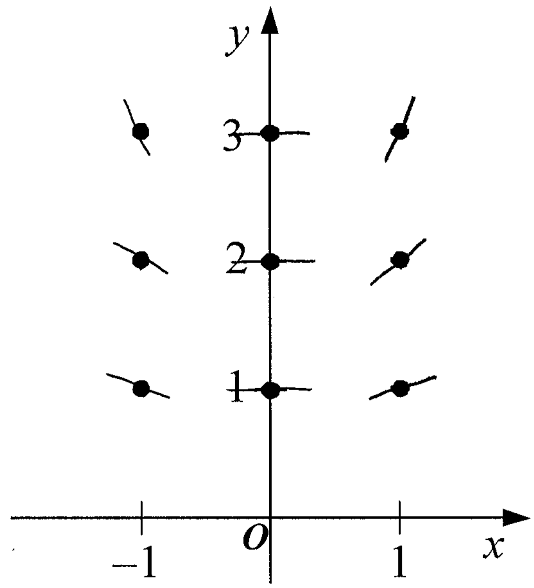
$$= \left| -\frac{1}{5}x \right| \overset{\text{want}}{<} 1$$

$\therefore \quad -5 < x < 5$

Thus radius of convergence is $\boxed{5}$

Superpod 3

Work for problem 4(a)



Work for problem 4(b)

i	x_i	y_i	y'	$dy = y' \cdot \Delta x$	Δx	$y_{i+1} = y_i + dy$	$x_{i+1} = x_i + \Delta x$
1	0	3	0	0	0.1	3	0.1
2	0.1	3	.15	.05	0.1	3.015	0.2

(0.2, 3.015)

$\boxed{f(0.2) \approx 3.015}$

Superpod 3

Work for problem 4(c)

$$\frac{dy}{dx} = \frac{xy}{2} \qquad f(0)=3$$

$$\int \frac{dy}{y} = \int \frac{x\,dx}{2} \qquad y=f(x)$$

$$\ln|y| = \frac{x^2}{4} + c \qquad e^c = c_1$$

only positive values of y

$$|y| = e^{\frac{x^2}{4}} \cdot c_1$$

$$3 = e^0 \cdot c_1$$

$$3 = c_1$$

$$y = 3e^{\frac{x^2}{4}}$$

$$y(0.2) = 3e^{\frac{0.04}{4}}$$

$$f(0.2) = \boxed{3e^{.01}}$$

Superpod 3

Work for problem 5(a)

At $t=5$ $m = 2 = r'$, $r = 30$

$\therefore$ $r - 30 = 2(t-5)$

$r = 2t + 20$ $r(5.4) = 10.8 + 20 = \boxed{30.8 \text{ ft}}$

Because the curve is concave down and thus has a decreasing slope, the approximation overestimates w/ a constant slope.

Work for problem 5(b)

$\frac{dv}{dt} = \frac{dr}{dt} \cdot \frac{dv}{dr}$ at $t=5$ $\frac{dv}{dr} = 4\pi r^2$

$= 2 \cdot 4\pi r^2 = 8\pi r^2$

at $t=5$, $r=30$

$= \boxed{7200\pi \ \frac{ft^3}{min}}$

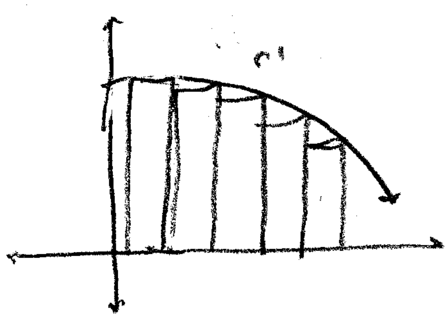
Superpod 3

Work for problem 5(c)

$$\int_0^{12} r'(t)\,dt = 4.0 \cdot 2 + 2 \cdot 3 + 1.2 \cdot 2 + .6 \cdot 4 + .5$$
$$= 8 + 6 + 2.4 + 2.4 + .5$$
$$= 14 + 4.8 + .5$$
$$= \boxed{19.3 \text{ ft}}$$

$\int_0^{12} r'(t)\,dt$ represents the total change in feet of the radius of the balloon from $t=0$ to $t=12$

Work for problem 5(d)



Becouse the curve is concave downward from $0 \le t \le 12$, the part c estimation is an understimate, because the right Reimann sum leaves out of the integral some areas under the curve.

Superpod 3

Work for problem 6(a)

$$\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$$

$$\int dx = \int \frac{1}{\sqrt{2t+1}}\, dt$$

$$x = (2t+1)^{\frac{1}{2}} + C$$

$$x(0) = -4$$

$$-4 = 1 + C$$

$$-5 = C$$

$$\boxed{x = (2t+1)^{\frac{1}{2}} - 5}$$

Work for problem 6(b)

$$y = x^3 - 3x$$

$$\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} - 3 \cdot \frac{dx}{dt}$$

$$= \boxed{3\left((2t+1)^{\frac{1}{2}} - 5\right)^2 \cdot \left((2t+1)^{-\frac{1}{2}}\right) - 3 \cdot (2t+1)^{-\frac{1}{2}}}$$

Superpod 3

Work for problem 6(c)

location at $t=4$,

$x(4) = (2\cdot 4+1)^{\frac{1}{2}} - 5 = -2$

$y = (-2)^3 - 3(-2) = -8+6 = -2$

$\boxed{(-2,-2)}$

speed at $t=4$,

$\frac{dy}{dt}$ at $t=4$ $= 3(3-5)^2\cdot(3) - 3\cdot(3)$

$= 12\cdot 3 - 3\cdot 3 = \underline{27}$

$\frac{dx}{dt}$ at $t=4$, $= \frac{1}{\sqrt{8+1}} = \underline{\frac{1}{3}}$

$V = \sqrt{\frac{1}{9} + 27^2} = \sqrt{\frac{6562}{9}}$

$= \boxed{\frac{\sqrt{6562}}{3}}$

= velocity