Superpod 2

Work for problem 1(a)

$\frac{dx}{dt} = 12t - 3t^2$

$\frac{dy}{dt} = \ln(1+(t-4)^4)$

$\frac{d^2x}{dt^2} = 12 - 6t$

$\frac{d^2y}{dt^2} = \frac{4(t-4)^3}{1+(t-4)^4}$

$\frac{d^2x}{dt^2}(2) = 12 - 6 \cdot 2 = 0$

$\frac{d^2y}{dt}(2) = \frac{4(2-4)^3}{1+(2-4)^4} = -\frac{32}{17}$

$\boxed{\text{acceleration vector: } \langle 0, -\frac{32}{17} \rangle}$

$\text{speed} = \sqrt{\left(\frac{dx}{dt}(2)\right)^2 + \left(\frac{dy}{dt}(2)\right)^2}$

$\text{speed} = \sqrt{12^2 + 2.833^2}$

$\boxed{\text{speed} = 12.330 \text{ units/sec}}$

Work for problem 1(b)

$P = (3, y)$

$y = 5 + \int_0^2 \ln(1+(t-4)^4)\,dt$

$\boxed{y = 13.671}$

Superpod 2

Work for problem 1(c)

$y - 13.671 = \frac{dy}{dx}(2) \cdot (x-3)$ $\qquad \frac{dy}{dx}(2) = \dfrac{\frac{dy}{dt}(2)}{\frac{dx}{dt}(2)} = \frac{2.833}{12} = 0.236$

$\boxed{y - 13.671 = 0.236 \cdot (x-3)}$

Work for problem 1(d)

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \overset{set}{=} 0$

$t = 4$

The expression above represents the speed of the object at time t. At $t=4$ the speed $= 0$ so the object is at rest at time $t = 4$.

Superpod 2

Work for problem 2(a)

$W(t) = 95(\sqrt{t})\sin^2(\frac{t}{6})$ g/hr.

$R(t) = 275\sin^2(\frac{t}{3})$ g/hr.

$t=15$

$W(15) \approx 131.78$

$R(t) \approx 252.87$

$W(t) - R(t)$ is negative

$\boxed{\text{Decreasing at } t=15}$

Work for problem 2(b)

$1200 + \int_0^{18} W(t)\,dt - \int_0^{18} R(t)\,dt$

$\approx \boxed{1309.788 \text{ gallons}}$

Superpod 2

Work for problem 2(c)

$W(t) - R(t) = 0$

$95\sqrt{t}\,\sin^2\left(\frac{t}{6}\right) - 275\sin^2\left(\frac{t}{3}\right) = 0$

$t \approx 6.495, 12.975$

* must test endpoints also *

t (hours)	$\int_0^t [W(t) - R(t)]\,dt$ (gallons)
0	1200
6.495	5
12.975	
18	

Work for problem 2(d)

$$0 = 1200 + \int_0^{18} 95\sqrt{t}\,\sin^2\left(\frac{t}{6}\right)dt - \int_0^{k} 275\sin^2\left(\frac{t}{3}\right)dt$$

Work for problem 3(a)

horizontal tangent at $x=0 \Rightarrow f'(0)=0$

$f^{(2)}(0) = \dfrac{(-1)^3 \cdot 3!}{5^2 \cdot 1^2} = \dfrac{-6}{25} < 0 \Rightarrow$ concave-down at $x=0$

$\therefore$ relative maximum at $x=0$ by second derivative test

Work for problem 3(b)

$$\frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$= \boxed{6 + 0 - \frac{\left(\frac{6}{25}\right)}{2!}x^2 + \frac{\left(\frac{6}{125}\right)}{3!}x^3}$$

Superpod 2

Work for problem 3(c)

$$\lim_{n\to\infty}\left|\frac{t_{n+1}}{t_n}\right| = \lim \left|\frac{\left(\frac{(-1)^{n+2}\cdot(n+2)!\cdot x^{n+1}}{5^{n+1}\cdot n^2\cdot(n+1)!}\right)}{\left(\frac{(-1)^{n+1}(n+1)!\cdot x^{n}}{5^{n}\cdot(n-1)^2\cdot n!}\right)}\right|$$

$$= \lim \left|(-1)\cdot x\cdot\frac{1}{5}\cdot\frac{(n-1)^2}{n^2}\cdot\frac{n+2}{n+1}\right|$$

$$= \frac{|x|}{5}\lim\left(\frac{n^2-2n+1}{n^2}\right)\cdot\lim\left(\frac{n+2}{n+1}\right)$$

$$= \frac{|x|}{5} < 1$$

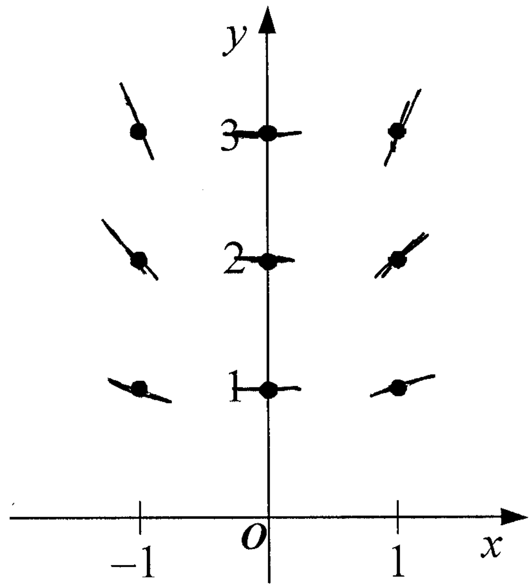
$$-1 < \frac{x}{5} < 1$$

$$-5 < x < 5$$

radius of conv. $= 5$

Superpod 2

Work for problem 4(a)



Work for problem 4(b)

i	x_i	y_i	$y'(x_i)$	dy	y_{i+1}
0	0	3	0	0	3
1	.1	3	.15	.015	3.015
2	.2	3.015			

$f(0.2) = 3.015$

Superpod 2

Work for problem 4(c)

$$\frac{dy}{y} = \frac{x}{2}\,dx$$

$$\ln|y| = \frac{1}{4}x^2 + c_1$$

$$e^{\frac{1}{4}x^2 + c_1} = \pm y$$

$$y = e^{\frac{x^2}{4}} e^{c_1}$$

$$y = Ce^{\frac{x^2}{4}}$$

init. cond : $f(0) = 3$

$$3 = Ce^{\frac{0^2}{4}}$$

$$3 = Ce^{0}$$

$$3 = C$$

$$y = 3e^{\frac{x^2}{4}}$$

at $x = .2$,

$$y = 3e^{\frac{(.2)^2}{4}}$$

$$f(.2) = 3e^{\frac{.04}{4}}$$

$$f(.2) = 3e^{.01}$$

Superpod 2

Work for problem 5(a)

$r(5.4) \approx r(5) + r'(5) \cdot (5.4 - 5.0)$

$\approx 30 + 2.0(0.4)$

$\approx 30 + 0.8$

≈ 30.8 ft

Work for problem 5(b)

$V = \frac{4}{3} \pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (30 \text{ ft})^2 (2.0 \text{ ft/min})$

$\frac{dV}{dt} = 7200\pi \text{ ft}^3/\text{min}$

Work for problem 5(c)

$\int_0^{12} r'(t)\,dt \approx (2-0)(4.0)+(5-2)(2.0)+(7-5)(1.2)+(11-7)(0.6)$
$+(12-11)(0.5)$

$\approx 2(4)+3(2)+2(1.2)+4(0.6)+1(0.5)$

$\approx 8+6+2.4+2.4+0.5$

$\approx 14+4.8+0.5$

$\approx 14+5.3 \approx 19.3$ ft

$\int_0^{12} r'(t)\,dt$ gives us the number of feet the radius of the balloon changes by from time $t=0$ to $t=12$ minutes.

Work for problem 5(d)

Greater

r is concave down on $(0,12)$ and r is increasing on this interval because $r'>0$.

∴ The Riemann sum is an upper sum because r is increasing at a decreasing rate.

Superpod 2

Work for problem 6(a)

$$\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$$

$$\int dx = \int \frac{1}{\sqrt{2t+1}}\, dt$$

$$x = \int (2t+1)^{-1/2}\, dt$$

$$x = (2t+1)^{-1/2} + C$$

$$x(t) = \sqrt{2t+1} + C$$

$$x(0) = -4 = \sqrt{2(0)+1} + C$$

$$-4 = 1 + C$$

$$C = -5$$

$$x(t) = \sqrt{2t+1} - 5$$

Work for problem 6(b)

$$y = x^3 - 3x$$

$$\frac{dy}{dx} = 3x^2 - 3$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{3x^2-3}{\sqrt{2t+1}}$$

$$= \frac{3x^2-3}{\sqrt{2t+1}}$$

$$\frac{dy}{dt} = \frac{3(\sqrt{2t+1} - 5)^2 - 3}{\sqrt{2t+1}}$$

Superpod 2

Work for problem 6(c)

$$\frac{dy}{dt} = \frac{3(\sqrt{2t+1}-5)^2 - 3}{\sqrt{2t+1}}$$

@ $t=4$, $\frac{dy}{dt} = \frac{3(-2)^2-3}{3} = 4-1 = 3$

$$\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$$

@ $t=4$, $\frac{dx}{dt} = \frac{1}{\sqrt{2(4)+1}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

velocity: $\langle \frac{1}{3}, 3 \rangle$

speed $= \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \sqrt{\frac{1}{9}+9} = \sqrt{82/9}$

$= \frac{1}{3}\sqrt{82} = \sqrt{82}/3$

$x(t) = \sqrt{2t+1} - 5$

$x(4) = \sqrt{9} - 5$

$= 3-5$

$= -2$

$y(t) = (\sqrt{2t+1}-5)^3 - 3(\sqrt{2t+1}-5)$

$y(4) = (-2)^3 - 3(-2)$

$= -8+6 = -2$

location: $(-2,-2)$