Superpod 1

Work for problem 1(a)

$$\frac{dx}{dt} = 12t - 3t^2 \qquad \frac{dy}{dt} = \ln\left(1+(t-4)^4\right)$$

(a) $\vec{v}(t) = \left\langle 12t - 3t^2, \ln\left(1+(t-4)^4\right) \right\rangle$

$\vec{a}(t) = \left\langle 12 - 6t, \frac{4(t-4)^3}{1+(t-4)^4} \right\rangle$

$\boxed{\vec{a}(2) = \left\langle 0, \frac{-32}{17} \right\rangle}$

Speed $= S(t) = \sqrt{(12t-3t^2)^2 + \ln\left(1+(t-4)^4\right)}$

$S(2) = \sqrt{144 + \ln^2 17}$

$\boxed{\approx 12.330}$

Work for problem 1(b)

$P(3, y_P)$ at $t = 2$

$y(0) = 5$

$y'(t) = \ln\left(1+(t-4)^4\right)$

$\int_0^2 y'(t)\,dt + y(0) = y(2)$

$\boxed{13.671 \approx y(2)}$

Work for problem 1(c)

line tangent at $P(3, 13.671)$

slope

$$m(t) = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln(1+(t-4)^4)}{12t-3t^2}$$

$$m(2) = \frac{\ln 17}{12}$$

point slope

$$y - 13.671 = \frac{\ln 17}{12}(x-3)$$

$$y = 0.236x - 0.708 + 13.671$$

$$\boxed{y = 0.236x + 12.963}$$

Work for problem 1(d)

object at rest when $\vec{v}(t) = \langle 0, 0 \rangle$

$\ln(1+(t-4)^4) = 0$ and $12t - 3t^2 = 0$

$1 + (t-4)^4 = 1$

$t(12-3t) = 0$

$(t-4)^4 = 0$

$t = 0$ $t = 4$

$t = 4$

$\boxed{\therefore \text{ at } t=4\text{, the object is at rest}}$

Work for problem 2(a)

1200 gal @ $t=0$

Because $W(t)$ $\frac{gal}{hr}$ are being pumped into the tank and $R(t)$ $\frac{gal}{hr}$ are being pumped out, $W(t)-R(t)$ will give the net rate of pumping.

$W(t)-R(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) - 275\sin^2\left(\frac{t}{3}\right)$ $\frac{gal}{hr}$

$W(15)-R(15) = 95\sqrt{15}\sin^2\left(\frac{15}{6}\right) - 275\sin^2\left(\frac{15}{3}\right)$ $\boxed{\approx -121.090 \frac{gal}{hr}}$

The amt. of water is decreasing, because $R(15) > W(15)$, meaning more water is being pumped out than being pumped in at $t=15$ hr.

Work for problem 2(b)

$$\text{Amt. water }(18) = 1200 + \int_0^{18} \left[95\sqrt{t}\sin^2\left(\tfrac{t}{6}\right) - 275\sin^2\left(\tfrac{t}{3}\right)\right]dt$$

$$= 1200 + 109.788\ldots = \boxed{1310 \text{ gal}}$$

Work for problem 2(c)

Min. $f(t)$ on $t \in [0, 18]$ min when f' $\ominus \to \oplus$

$W(t) - R(t) = 0$

$t \in \{ 6.495, 12.975, \}$

but only go from $\ominus$ to $\oplus$ at $t \in \{4.0323\ldots, 15.0867\ldots\}$

$1200 + \int_0^0 (W(t) - R(t))\,dt = 1200$

$1200 + \int_0^{6.495} (W(t) - R(t))\,dt = 525.2422$

$1200 + \int_0^{12.975} (W(t) - R(t)\,dt) =$

$1200 + \int_0^{18} (W(t) - R(t))\,dt \approx 1310$

$t = 6.495$, water is at abs. min.

Work for problem 2(d)

$$1200 + \int_0^{18} 95\sqrt{t}\,\sin^2\left(\tfrac{t}{6}\right)dt - \int_0^{k} 275\sin^2\left(\tfrac{t}{3}\right)dt$$

Work for problem 3(a)

Because the tangent line is horizontal at $x=0$, $f'(0)=0$

$$f''(0) = \frac{(-1)^3\ (2+1)!}{5^2(2-1)^2} = \frac{-1\cdot 6}{25} = \frac{-6}{25} < 0$$

$\therefore$ By second derivative test, $f(0)$ is a relative maximum

Work for problem 3(b)

$$f(0) = f(0) + \frac{f'(0)\ (x-a)}{1!} + \frac{f''(0)\ (x-a)^2}{2!} + \frac{f'''(0)\ (x-a)^3}{3!} + \dots$$

$$= 6 + \frac{0(x)}{1} + \frac{\left(\frac{-6}{25}\right)\cdot x^2}{2} + \frac{\frac{6}{125}\cdot x^3}{6} + \dots$$

$$\boxed{= 6 - \frac{3x^2}{25} + \frac{x^3}{125} - \dots}$$

Work for problem 3(c)

$$L \overset{\text{set}}{=} \lim_{n\to\infty} \left| \frac{\dfrac{(-1)^{n+2}(n+2)!\cdot x^{n+1}}{5^{n+1}(n^2)\cdot(n+1)!}}{\dfrac{(-1)^{n+1}(n+1)!\cdot x^n}{5^n\cdot(n-1)^2\cdot n!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+2)\cdot(n-1)^2\cdot x}{5\cdot n^2\cdot(n+1)} \right|$$

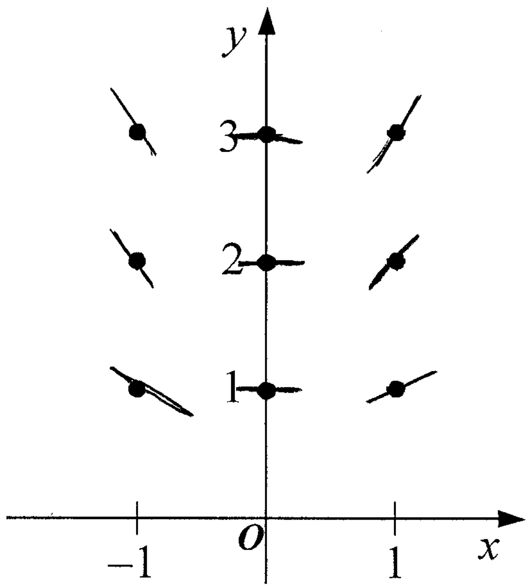
$$= \left|\frac{x}{5}\right| \lim_{n\to\infty} \left| \frac{n+2\cdot(n-1)^2}{n^2(n+1)} \right| = \left|\frac{x}{5}\right|$$

$$L \overset{\text{set}}{<} 1 \Rightarrow \left|\frac{x}{5}\right| < 1$$

$-5 < x < 5$ Therefore, the radius of convergence is 5

Superpod 1

Work for problem 4(a)



x	y	$\frac{xy}{2} = \frac{dy}{dx}$
-1	1	$-\frac{1}{2}$
-1	2	-1
-1	3	$-\frac{3}{2}$
0	1	0
0	2	0
0	3	0
1	1	$\frac{1}{2}$
1	2	1
1	3	$\frac{3}{2}$

Work for problem 4(b)

$\Delta x = 0.1$

i	x_i	y_i	Δx	$dy = \frac{dy}{dx} \cdot \Delta x = \frac{xy\Delta x}{2}$	$x_{i+1} = x + \Delta x$	$y_{i+1} = y + dy$
1	0	3	0.1	$\frac{0 \cdot 3 \cdot 0.1}{2} = 0$	0.1	$3 + 0 = 3$
2	0.1	3	0.1	$\frac{0.1 \cdot 3}{2}(0.1) = 0.015$	0.2	$3 + 0.015 = 3.015$
3	0.2	3.015				

$\boxed{f(0.2) \approx 3.015}$

Superpod 1

Work for problem 4(c)

$f(0) = 3$

$$\frac{dy}{dx} = \frac{xy}{2}$$

$$xy\,dx = 2\,dy$$

$$\int \frac{x\,dx}{2} = \int \frac{dy}{y}$$

$$\frac{x^2}{4} + C = \ln|y|$$

$$e^{\frac{x^2}{4}+C} = y$$

$$C_1 e^{\frac{x^2}{4}} = y$$

$$C_1 e^{\frac{0}{4}} = 3$$

$$C_1 \cdot 1 = 3$$

$$C_1 = 3$$

$$y = 3e^{\frac{x^2}{4}}$$

$$\boxed{y(0.2) = 3e^{\frac{0.2^2}{4}} \approx 3.030}$$

Work for problem 5(a)

$R(5) + \frac{dy}{dx} \cdot dx = R(5.4)$

$30 + 2(.4) = 30.8$ feet

This value above is greater than the true value because r is concave downward on the interval $0 < T < 12$, so ~~a tan~~ using a linear approximater will provide an overestimate.

Work for problem 5(b)

$r = 30$ at $t = 5$

$V = \frac{4}{3} \pi r^3$

$V' = 4 \pi r^2 r' \Rightarrow$ insert $r = 30$

$36{,}000 \pi V' = 3600 \pi r' \quad r' = 2$

$36{,}000 \pi V' = 7200 \pi$

$V' = \frac{1}{5} \text{ feet}/\text{min}^3$

Work for problem 5(c)

$\int_0^{12} r'(t)\,dt = 2.4\left(\frac{1}{2}r'(2) + r'(5) + r'(7) + r'(11) + .5\,r'(12)\right)$

$2.4(2 + 2 + 1.2 + .6 + .25)$

$2.4(6.05) = 14.52$ feet.

The value above represents what the radius of the balloon is after twelve minutes have elapsed

Work for problem 5(d)

My approximation in part c. is less than $\int_0^{12} r'(t)\,dt$ because the graph is concave downward on the interval $0 \le t \le 12$, so a right Riemann sum will produce an underestimate of the true value.

Work for problem 6(a)

Find $x(t)$

$$x'(t) = (2t+1)^{-1/2}$$

$$x(t) = \tfrac{1}{2} \cdot 2(2t+1)^{1/2}$$

$$x(t) = (2t+1)^{1/2} + C$$

$$-4 = \sqrt{1} + C$$

$$C = -5$$

$$\boxed{x(t) = (2t+1)^{1/2} - 5}$$

Work for problem 6(b)

Find $y'(t)$ $\quad x = (2t+1)^{1/2} - 5$

$$y(t) = \left((2t+1)^{1/2} - 5\right)^3 - 3\left((2t+1)^{1/2} - 5\right)$$

$$\boxed{y'(t) = 3\left((2t+1)^{1/2} - 5\right)^2 (2t+1)^{-1/2} - 3(2t+1)^{-1/2}}$$

Work for problem 6(c)

Location of the particle $= (x(t), y(t))$

$x(t) = (2t+1)^{1/2} - 5$

$x(t) = -2$

$y = x^3 - 3x = -2$

$\boxed{(-2,-2) = \text{location}}$

Speed: $\sqrt{dx^2 + dy^2}$

$dx = 1/3$

$dy = 3(-2)^2 \, 1/3 - 3 \cdot (1/3) = 3$

Speed $= \sqrt{(1/3)^2 + (3)^2}$

$\boxed{= 3.01846}$