Mr. Hansen
HappyCal HW due 9/25/08
[excerpt]

§3-4 #36.

Given: $f(x) = x^5 = y$

Find: $f'(c)$

[Note: clearly, $f'(c) = \left.\frac{dy}{dx}\right|_{x=c} = \left.5x^4\right|_{x=c} = 5c^4$. Our task is to prove this rigorously.]

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c} \quad \text{by def. of deriv. at a point}$$

$$= \lim_{x\to c} \frac{x^5 - c^5}{x - c} \quad \text{by subst.}$$

$$= \lim_{x\to c} \frac{(x-c)(x^4 + x^3c + x^2c^2 + xc^3 + c^4)}{x-c} \quad \text{by the hint}$$

$$= \lim_{x\to c} (x^4 + x^3c + x^2c^2 + xc^3 + c^4) \quad \text{by nonzero cancellation}$$

$$= \lim_{x\to c} x^4 + \lim_{x\to c} x^3c + \lim_{x\to c} x^2c^2 + \lim_{x\to c} xc^3 + \lim_{x\to c} c^4$$

by limit property (limit of a sum)

$$= c^4 + c\lim_{x\to c} x^3 + c^2\lim_{x\to c} x^2 + c^3\lim_{x\to c} x + \lim_{x\to c} c^4$$

by cont. of the power fcn. $x^4$, limit property involving const. mult. of a fcn.

$$= c^4 + c(c^3) + c^2(c^2) + c^3(c) + \lim_{x\to c} c^4$$

by cont. of the power fcns. $x^3$, $x^2$, and the limit of identity fcn.

$$= c^4 + c^4 + c^4 + c^4 + c^4 \quad \text{by algebra, limit of a const.}$$

$$= 5c^4 \quad \text{as expected } \square$$

Whew! Henceforth we will almost never compute derivatives this way (i.e., by using the definition and limit properties). Instead, we will develop "templates" for each category of fcn. (polynomial, exponential, etc.) and rules for handling all types of combinations.