Algebra II / Mr. Hansen
5/4/2016

Name: KEY (Block 2)

. . . .

Test on Vec/Trig/Prob/Stat

Instructions: Graphing calculator and PENCIL are required . If you need more room for rough scratch work, use the reverse sides, which will not be graded (unless you need more room for work that is to be graded, in which case you should write OVER and continue on the back side). Circle all answers in problems involving work (no need to circle if the answer is a fill-in without any work).

Note: For this test, please round all approximate answers to 4 decimal places after the decimal point unless otherwise stated.

Part I: Short Answer (4 pts. each, except for the first one, which is scored pass-fail)

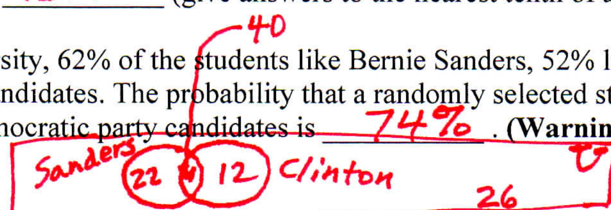
1. If the answer to a probability or expected-value question (e.g., the expected number of people who will be able to board an aircraft within the first 5 minutes of boarding) is *really* important in the world of business, how should you go about finding an answer? Run a simulation!

2,3. In how many ways can the 52 cards of a standard deck be arranged? Order matters, obviously. Give both an exact answer (using appropriate symbols) and an approximate answer (to the nearest power of 10).
Exact (symbolic): ${}_{52}P_{52}$ or $52!$ Approximate answer: 10 to the 68 power

4. The law of sines can sometimes run into a problem with ambiguous cases. However, if the unknown angle is known to be the second-largest or third-largest angle in the triangle, the law of sines is safe to use, because there is no possibility that the unknown angle could be obtuse .

5,6. If the unknown angle in a triangle has a sine of 0.818, then the angle could be either $54.9°$ or $125.1°$ (give answers to the nearest tenth of a degree).

7. At a large university, 62% of the students like Bernie Sanders, 52% like Hillary Clinton, and 40% like both candidates. The probability that a randomly selected student supports at least one of these two Democratic party candidates is 74% . **(Warning: Instant failure if you say 114%.)**



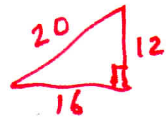
8. Let **u** = <4, 6>, and let **v** = <–3, 6>. Find the magnitude of the vector that connects the terminal point of vector **u** to the terminal point of vector **v**. No work is required. Answer: 7

9. The other word we used for magnitude (of a vector) is norm (also acceptable: length)

10,11. Two vectors (neither of which is <0, 0>) are said to be *perpendicular* (or *orthogonal*) if and only if their dot product is zero. Compute the dot product: <8, –2> · <–3, –11> = $-24 + 22 = -2$
Are these two vectors perpendicular? NO

12. Find the area of a triangle whose sides have lengths 12 ft., 16 ft., and 20 ft. Work is optional, but be sure to circle your answer.

Note: Triangle is right, since sides have a 3:4:5 ratio.



$A = \frac{1}{2}bh = \frac{1}{2}(16)(12) = \boxed{96 \text{ ft}^2}$

Part II. Free Response (12 pts. per numbered problem). Work is required for full credit. Give all approximate answers in a decimal format, rounded to 4 decimal places after the decimal point.

13. An urn contains 12 identically sized balls, 6 of which are black and 6 of which are red. Those are the only balls present in the urn.

(a) Compute the probability of obtaining two reds and a black (in that order) when balls are drawn randomly, without replacement.

$$P(R_1 \cap R_2 \cap B_3) = \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10} = \frac{3}{22} = 0.13\overline{63}$$

(b) Compute the probability of obtaining at least one red ball when 3 balls are drawn randomly, without replacement.

$$P(\text{at least one red}) = 1 - P(\text{no reds}) = 1 - \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{4}{10}$$

$$= 1 - \frac{1}{11} = \frac{10}{11} = 0.\overline{90}\overline{90}$$

(c) Compute the probability of obtaining two reds and a green (in that order) when balls are drawn randomly, *with* replacement.

There are no greens, so 0.

14. Four cards are drawn, without replacement, from a standard 52-card deck that has been shuffled well.

(a) Compute the probability of obtaining all 4 aces.

$$P(4 \text{ aces}) = \frac{\binom{4}{4}}{\binom{52}{4}} = \frac{1}{270{,}725} \approx 0.000003694$$

(b) Compute the probability of obtaining at least one ace.

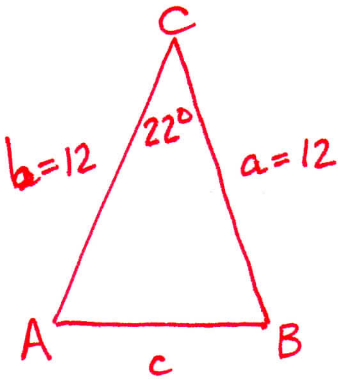
$$P(\text{at least one ace}) = 1 - P(\text{no aces}) = 1 - \frac{\binom{48}{4}}{\binom{52}{4}}$$

$$= 1 - \frac{194{,}580}{270{,}725} \approx 0.2813$$

(c) Compute the probability of obtaining 2 aces and 2 jacks.

$$P(2 \text{ aces and } 2 \text{ jacks}) = \frac{\binom{4}{2}\binom{4}{2}}{\binom{52}{4}} = \frac{36}{270{,}725} \approx 0.000133$$

15. Below, make a reasonably neat sketch of triangle ABC that has the following given information: $a = 12$, $b = 12$, $\angle C = 22°$. (Angle C is what your textbook commonly called γ.) Then solve the triangle. Put rough sketch work on the back of one of the other sheets. What you show below should be clean and coherent. Sides must be correct to 4 decimal places after the decimal point, and angles should be given in the format xx°xx.x', where the ' symbol denotes minutes.



Since ΔABC is isosceles, $\angle A \cong \angle B$. Thus we have

$$\boxed{\begin{aligned}\angle A &= 79°00.0' \\ \angle B &= 79°00.0'\end{aligned}}$$

To find c, use Law of Sines:

$$\frac{\sin \angle C}{c} = \frac{\sin \angle A}{a}$$

$$\frac{\sin 22°}{c} = \frac{\sin 79°}{12}$$

$$c = \frac{12 \sin 22°}{\sin 79°} \approx 4.5794$$

16. Sketch vectors **u** = <4, –3>, **v** = <–3, 4>, and the vector **u** + **v**, all on the same set of axes. (Make your sketch down below, after the questions. Make your sketch medium-sized, so that your middle-aged teacher with his weak eyes will be able to grade it.) $\vec{u}+\vec{v} = \langle 4,-3\rangle + \langle -3,4\rangle$

$= \langle 1,1\rangle$

(a) Label each vector.

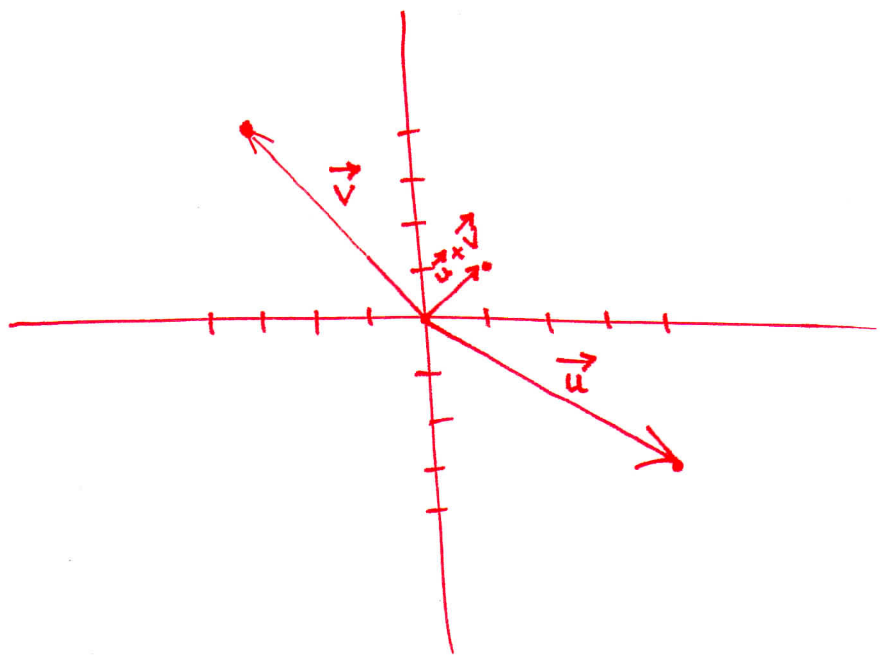
(b) Compute the vector 2**u** – 3**v**. No need to sketch it, but for full credit you must begin by writing 2**u** – 3**v** = . . . Remember to circle your answer. Show enough work so that it is clear that you know what you are doing.

$$2\vec{u}-3\vec{v} = 2\langle 4,-3\rangle - 3\langle -3,4\rangle = \langle 8,-6\rangle - \langle -9,12\rangle$$

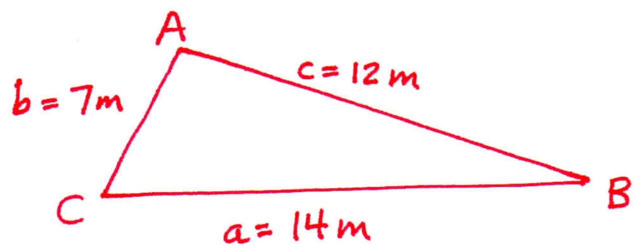
$$= \langle 17,-18\rangle$$

(c) Write two correct notations for the zero vector. (The zero vector is the vector that, when added to another vector, does not change it.)

Notation 1: $\vec{0}$ Notation 2: $\langle 0,0\rangle$



17. Solve the triangle whose sides have lengths 14 m, 7 m, and 12 m, and find the area to the nearest tenth of a square meter. Give angles to the nearest tenth of a degree. A sketch is required.



$a^2 = b^2 + c^2 - 2bc \cos \angle A$

$14^2 = 7^2 + 12^2 - 2(7)(12) \cos \angle A$

$196 = 49 + 144 - 168 \cos \angle A$

$3 = -168 \cos \angle A$

$-\frac{3}{168} = \cos \angle A$

$\angle A \approx 91.0^\circ$

$b^2 = a^2 + c^2 - 2ac \cos \angle B$

$49 = 196 + 144 - 2(14)(12) \cos \angle B$

$-291 = -336 \cos \angle B$

$\frac{291}{336} = \cos \angle B$

$\angle B \approx 30.0^\circ$

$c^2 = a^2 + b^2 - 2ab \cos \angle C$

$144 = 196 + 49 - 2(14)(7) \cos \angle C$

$-101 = -196 \cos \angle C$

$\frac{101}{196} = \cos \angle C$

$\angle C \approx 59.0^\circ$

Check: angles add to 180° ✓

Area by Hero's Formula:

$s = \frac{a+b+c}{2} = \frac{14+7+12}{2} = 16.5$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16.5(16.5-14)(16.5-7)(16.5-12)} \approx 42.0 \text{ m}^2$