ALGEBRA II MIDTERM REVIEW

Topics Covered

Chapter 0
Sets of numbers
Integer and rational exponents
Polynomials and operations
Factoring polynomials
Rational expressions
Radicals
Complex numbers

Chapter 1
Solving equations – linear, quadratic, radical, abs. value
Interval notation
Solving inequalities – linear, quadratic, abs. value
Word problems – working together, mixture, rate-distance-time
Chart of signs (finding signs of overall product or quotient depending on signs in each interval)

Chapter 2
Graphing basics: distance and midpoint
Graphing equations: point-plotting, intercepts
Graphing lines
Linear word problems

Chapter 3
Definition of a function: domain, range
Graphs of functions – increasing/decreasing
Graphing piecewise functions
Graphing techniques – transformations
Function composition
Inverse functions
Direct and inverse variation

Chapter 4
Graphing parabolas – finding vertex and intercepts
Graphing polynomials of higher degree
Long division and synthetic division
Factoring & finding the zeros – the Rational Root Theorem
Finding complex zeros
Quadratic word problems – finding the max/min
Graphing rational functions (horizontal and vertical asymptotes)

CHAPTER 0 REVIEW

1. Simplify: $(2x-3y)^3$

2. Simplify: $\left(\dfrac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$

3. Simplify the following:

   a) $\sqrt{48x^3y^8z^9}$

   b) $9^{\frac{3}{2}}+27^{-\frac{2}{3}}$

4. Simplify the following:

   a) $\dfrac{6}{8+\sqrt{2}}$

   b) $\dfrac{8}{\sqrt[3]{16}}$

5. Simplify: $\dfrac{4y^2-9}{2y^2+9y-18}\div\dfrac{2y^2+y-3}{y^2+5y-6}$

6. Simplify: $\dfrac{2}{x+3}-\dfrac{1}{x^2+7x+12}$

Factor completely for problems 7 and 8:

7. a) x^3+4x^2+x+4

   b) x^6-27

8. a) x^4-16

   b) $-2x^4+4x^2-2$

9. Simplify: $\left(\dfrac{x^{\frac{1}{4}}y^{\frac{-2}{3}}}{x^{\frac{-5}{4}}y^{\frac{1}{3}}}\right)^{-2}$

10. Simplify: $\dfrac{x^2+2x-3}{x^2-2x-3}\cdot\dfrac{3-x}{3+x}$

11. Simplify: $5x^2-2[2x-3(x+3)(4x-1)]$

12. Identify the following numbers as rational or irrational. You may have to simplify some of them first.

a) $0.\overline{3}+0.\overline{5}$

b) $\pi+\pi^{-1}$

c) $\dfrac{4}{\sqrt{2}}-\sqrt{8}$

d) $2.011223334\overline{4}$

13. Simplify: $\dfrac{x}{x^2+x-2}-\dfrac{2}{x^2-5x+4}$

14. Put into radical form and simplify: $(12)^{\frac{3}{2}}\cdot(27)^{\frac{1}{2}}$

CHAPTER 1 REVIEW

You may not use a calculator for the following problems.

1. Max has a 5 oz. sippy cup that is filled with 60% water and 40% orange juice. How many ounces should he pour out and replace with pure orange juice in order to make his final drink have 75% orange juice?

2. Emma took a trip from city A to city B, a total distance of 350 miles. She traveled part of the way on train (averaging 60 mph) and part of the way on the bus (averaging 40 mph). The entire trip took 6 hours. How many miles was each leg of the journey?

3. Sammy and Cosi can paint a room together in 6 hours. Working by himself, Sammy takes 2 hours less than Cosi to paint the room. How long do each of them take individually?

4. A train leaves Kyoto for Tokyo at noon traveling 240 mph. Ten minutes later, a train leaves Tokyo for Kyoto traveling 160 mph. If Kyoto and Tokyo are 300 miles apart, at what time will the trains pass each other?

5. Solve: $2x-3(x+2)=5-(5x-2)$

6. Solve: $5-2\sqrt[3]{x-1}=9$

7. Solve: $\frac{2}{3}x-\frac{1}{6}=-\frac{3}{2}x-2$

8. Solve: $\dfrac{2}{x+3}-\dfrac{3}{x}=\dfrac{8}{x(x+3)}$

9. Solve: $\dfrac{5}{y-3}+1=\dfrac{30}{y^2-9}$

10. Solve: $(x-2)(x-4)=(2x+1)(3x-2)$

11. Solve: $x^2-36=5x$

12. Solve: $\sqrt{2x+1}+x=7$

13. Solve: $\sqrt{2x+3}-\sqrt{x+1}=1$

14. Find two consecutive integers whose product is 5 less than 5 times their sum.

A calculator is permitted for problems 15 through 17.

15. Mary drove from A to B at a speed of 50 mph. On the way back, she drove at 60 mph. The total trip (there and back) took 4.4 hours of driving time. Find the distance between A and B.

16. A chemist has 12 liters of a 30% acid solution. What type of acid solution must she add to obtain a total 15 liters of a 35% solution? Give your answer as a percent.

17. Andrew (working alone) takes 5 days to paint a house, while Bill takes 8 days to paint the same house. However, if Andrew starts painting, by himself, for 2 days before Bill joins him, how long will it take the two of them working together to finish painting the house? (In other words, how many days are required in addition to the 2 days Andrew already spent?)

You may not use a calculator for the following problems. For each inequality, give your solution using interval notation.

18. Solve: $2-3(x+2)<5x-8$

19. Solve: $\dfrac{3-2x}{4}\geq -5$

20. Solve: $-2<5+\frac{1}{2}x<4$

21. Solve: $3-|1-2x|=-4$

22. Solve: $2x^2+x>1$

23. Solve: $\left|\dfrac{x+2}{3}\right|\leq 6$

24. Solve: $-2(2x-3)(2-x)(x^2+4)(x+1)^2>0$

25. Solve: $7+2|2x-1|\geq 3$

26. Solve: $(x-2)^2(x^2+4)\leq 0$

CHAPTER 2 REVIEW

You may use a calculator on the following problem.

1. Assume that the maximum speed your car will go is a linear function of the steepness of the hill it is going up or down. Suppose that the car can go a maximum of 55 mph up a 5° hill, and a maximum of 104 mph down a 2° hill. Going downhill can be thought of as going up a hill of -2°.

a) Write the equation expressing maximum speed (y) in terms of steepness (x).
b) How fast could you go down a 7° hill?
c) If your max speed is 83 mph, how steep is the hill?
d) Find the y-intercept and explain what it means in terms of the problem.
e) Find the x-intercept and explain what it means in terms of the problem.
f) Find the slope and explain what it means in terms of the problem.

NO CALCULATORS ON THE FOLLOWING PROBLEMS.

2. Graph the following equation: $y = -2|x+1|-1$

3. Explain in words how the graph in the previous question differs from the normal $y = |x|$ graph. *Note:* You should be able to answer this question even if you failed to make the graph itself.

4. Graph and label the following lines on the same axes:

$$L_1 : 3x - 6y = -9$$
$$L_2 : x = -\tfrac{1}{2}y + 3$$
$$L_3 : 2y - 3 = \tfrac{2}{3}x$$
$$L_4 : x = -1$$

5. Find the equation, in standard form, of the line that is perpendicular to $2x - 3y = -1$ and that shares a y-intercept with $5x + 4y = -20$.

6. Sketch the graph, labeling all *x*- and *y*-intercepts: $x^2 + y^2 = 9$

7. Consider the points A(-5, -2) and B(-10, 5).
   a) Find the midpoint of segment AB.
   b) Find the length of segment AB. Simplify the square root fully.

8. Graph the following equation: $y = 2\sqrt{-x} - 2$
   Also explain in words how this graph differs from the normal $y = \sqrt{x}$ graph.

9. Graph the equation: $y = -x^3 + 2$

10. Graph the triangle formed by the points A(2,0), B(-1, 4), and C(-3, -5). Find the equation of the median of the triangle drawn from point A to the opposite side. The median of a triangle is defined to be the line segment joining a vertex to the midpoint of the opposite side.

CHAPTER 3 REVIEW

1. Indicate which of the following are functions. If not, give an example of an input that shows that it is not. If it is, state the domain.

   a) $y^2 = x+1$

   b) $y = \dfrac{\sqrt{x}}{x+1}$

2. Draw a function for the domain and range listed below. There is more than one right answer.

   Domain: [–2,5] Range: (-5, 1)

3. Do all parts (a through f) of #35, p. 259.

4. Sketch the graph of the following function:

$$f(x) = \begin{cases} -x-1 & x < -1 \\ 2 & -1 \le x < 2 \\ (x-3)^2 & x > 2 \end{cases}$$

5. Write the equation of a parabola that has been flipped over the x-axis, moved three units up, and moved two units to the right.

6. Let $f(x) = 2 - x$ and $g(x) = -x^2 + 2x$. Evaluate the following:

   a) $f(-3)$ b) $g(3)$

   c) $f(g(1))$ d) $g(f(3))$

7. Using the same functions as in #6, find and simplify the following:

   a) $f \circ g$ b) $g \circ f$

8. Sketch a graph of the function $f(x) = \sqrt{x+1} - 2$. The only points you need to label are the vertex, *x*-intercept, and *y*-intercept.

9. Now find the inverse of the function in problem #8. Also sketch the inverse on the same set of axes.

You may use a calculator for the following problems.

10. The force needed to keep a car from skidding on a curve varies directly with the weight of the car, directly with the square of the speed of the car, and inversely with the radius of the curve.

    a) Write the general equation.
    b) It takes 3800 pounds of force to keep an 1800 pound car from skidding on a curve with radius 425 feet at 45 mph. Now find the value of *k* in your equation.
    c) Use your equation to find the force that is needed to keep the same car from skidding when it drives on a curve with radius 450 feet at 55 mph.

11. A typical long-distance calling plan from the 1980s consisted of a $30 base charge in addition to $.10 for each minute of long-distance calls.

    a) Write a function representing this plan.
    b) Find $f(180)$ and explain what it means in terms of the problem.
    c) Find the inverse of the function.
    d) Find $f^{-1}(100)$ and explain what it means in terms of the problem.

CHAPTER 4 REVIEW

1. Simplify the following. Write your answer in the form $a+bi$:
   a) $(3-5i)-(-2-i)$
   b) $(2-i)(-3+2i)$
   c) $\dfrac{1-2i}{3+3i}$

2. Use long division to divide the following: $\dfrac{2x^4-2x^2+3x-2}{x^2+x-3}$

3. The height above the ground (in feet) of a rocket after x seconds is given by $y=-16x^2+128x$.
   a) How high is the rocket after 2 seconds?
   b) After how many seconds does the rocket hit the ground?
   c) What is the maximum height the rocket reaches?
   d) How long does the rocket take to reach its maximum height?

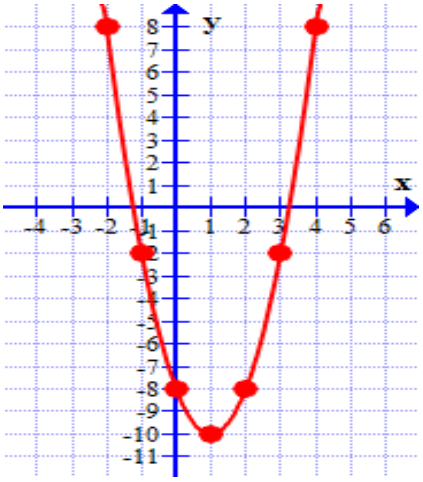
4. Find a polynomial with zeros of: $i,-i,2,-2$. Do NOT leave it in factored form; you must multiply it out.

5. Find an equation (any form acceptable) of a parabola with a vertex at (-4, 5) and a *y*-intercept at (0, -1).

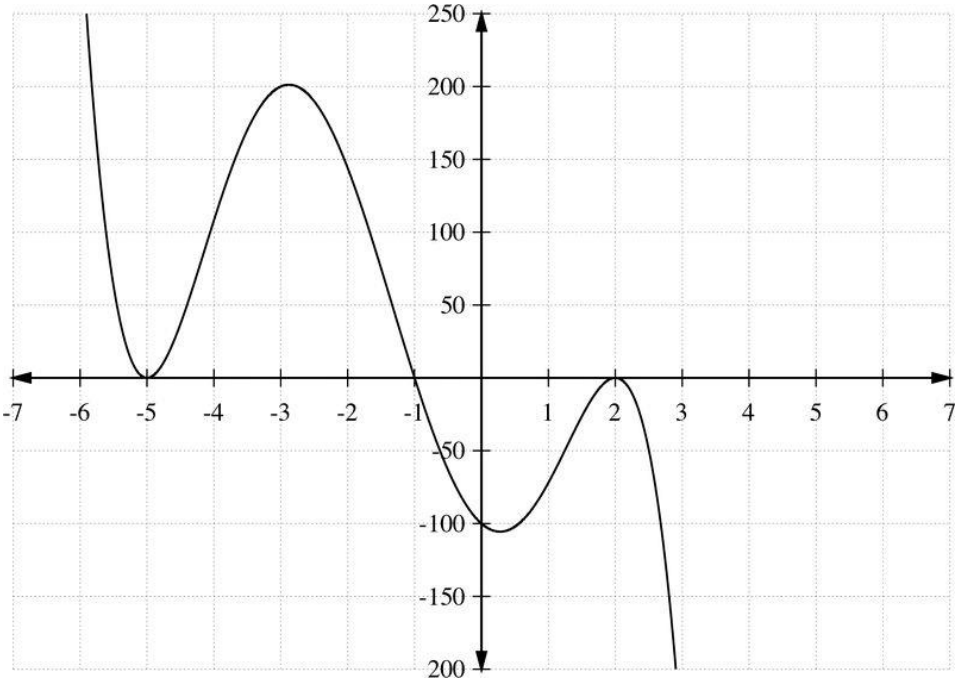
6. Sketch a graph of the function $y=-2x(x-2)^2(x+4)$. Be sure to approximate any real roots. Feel free to adjust the scale on either axis as necessary.

7. Sketch a graph of the function $y=\frac{1}{4}x^2-2x+1$. Find the vertex, y-intercept, and at least one other point.

8. Write an equation for the following parabola:



9. Write an equation for the following polynomial. You may leave it in factored form:



10. Consider the following polynomial: $x^4 - x^3 - 2x^2 + 6x - 4$. Factor this polynomial completely and find all zeros.

11. Graph $y = \dfrac{2x^2 + 1}{(x-3)(x+3)}$

12. Graph $y = \dfrac{2x - 2}{(x-3)(x+2)}$

13. Graph $y = \dfrac{-x^2}{(x-2)(x+2)^2}$

14. Graph $y = \dfrac{2x^2 - 2}{x^2 + 1}$