Mr. Hansen
Algebra II review problems
due: 5/4/2016

Review problems from Web page

1. See book for odd-numbered answers. Here are the even-numbered ones:

p.870 #60. $\vec{u}-\vec{v} = \langle 7,-2\rangle - \langle -4,5\rangle = \boxed{\langle 11,-7\rangle}$

p.870 #68. [Remember: Make a unit vector by dividing any vector by its norm. "Norm" is a synonym for "magnitude."]

$$\frac{\langle -11,60\rangle}{\|\langle -11,60\rangle\|} = \frac{\langle -11,60\rangle}{\sqrt{11^2+60^2}} = \frac{\langle -11,60\rangle}{61} = \boxed{\left\langle \frac{-11}{61}, \frac{60}{61}\right\rangle}$$

2. (a) $P(3 \text{ heads}) = P(H_1 \cap H_2 \cap H_3) = P(H_1)\cdot P(H_2)\cdot P(H_3)$ [WARNING: VALID ONLY BECAUSE COIN FLIPS ARE INDEPENDENT!]

$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \boxed{\frac{1}{8}}$

(b) $P(\geq 1 \text{ head}) = 1 - P(\text{no heads}) = 1 - P(T_1 \cap T_2 \cap T_3)$

$= 1 - \frac{1}{8}$ ← [by reasoning similar to (a)]

$= \boxed{\frac{7}{8}}$

(c) $P(H_1 H_2 T_3) = P(H_1 \cap H_2 \cap H_3) = P(H_1)\cdot P(H_2)\cdot P(T_3)$

$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \boxed{\frac{1}{8}}$

NOTE: If order did not matter, there would be 3 ways to obtain 2 heads and 1 tail, and the probability would be $\frac{3}{8}$, not $\frac{1}{8}$.

3. (a) $P(5 \text{ face cards in a row}) = \frac{\binom{12}{5}}{\binom{52}{5}} = \frac{792}{2{,}598{,}960} \approx \boxed{0.0003047}$

(b) $P(\geq 1 \text{ face card}) = 1 - P(\text{no face cards}) = 1 - \frac{\binom{40}{5}}{\binom{52}{5}}$

$= 1 - \frac{658{,}008}{2{,}598{,}960} = 1 - 0.25318\ldots \approx \boxed{0.7468}$

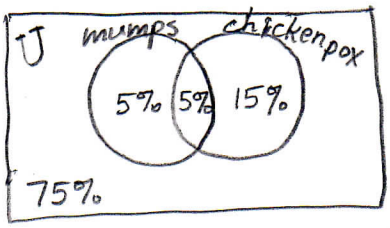
(c) $P(\geq 1 \text{ ace}) = 1 - P(\text{no aces}) = 1 - \frac{\binom{48}{5}}{\binom{52}{5}} = 1 - \frac{1{,}712{,}304}{2{,}598{,}960} \approx \boxed{0.3412}$

4. (a) $P(\text{mumps})\cdot P(\text{chicken pox}) = 0.1\cdot 0.2 = 0.02$

But... that is <u>not</u> the same as $P(\text{mumps} \cap \text{chicken pox})$, which was given to be 0.05. Since the joint probability (i.e., the probability of the intersection) does not equal the product of the individual probabilities, the events are **not independent.**

[Common student error is to say $P(\text{mumps} \cap \text{chicken pox}) = P(\text{mumps})\cdot P(\text{chicken pox})$, but that is <u>not</u> true in general. What is always true is that $P(A \cap B) = P(A)\cdot P(B|A)$.]

(b)



P(neither disease) = 75% = 0.75

(c) P(at least one disease) = P(mumps ∪ chicken pox)
= P(mumps) + P(chicken pox) − P(mumps ∩ chicken pox)
= 10% + 20% − 5%
= 25% = 0.25

ALT. METHOD: P(at least one disease) = 1 − (answer to part (a))
= 1 − 0.75 = 0.25

(d) P(mumps ∪ chicken pox), using the union symbol

5.

day	height (in.)
1	8
3	16
4	24
5	26
7	29
8	35
9	39
11	41

(a) sample mean = $\bar{y}$ [note: shown as $\bar{x}$ on your calculator's STAT CALC 1]

$\bar{y}$ = 27.25 in.

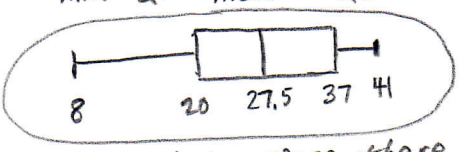
mode = undefined

median = Q2 = 27.5

sample s.d. = $s \approx$ 11.33578

(b) 5-number summary [by calc.]: 8, 20, 27.5, 37, 41

↑ min ↑ Q1 ↑ median ↑ Q3 ↑ max



[Note: This modified boxplot has no dots, since there are no outliers.]

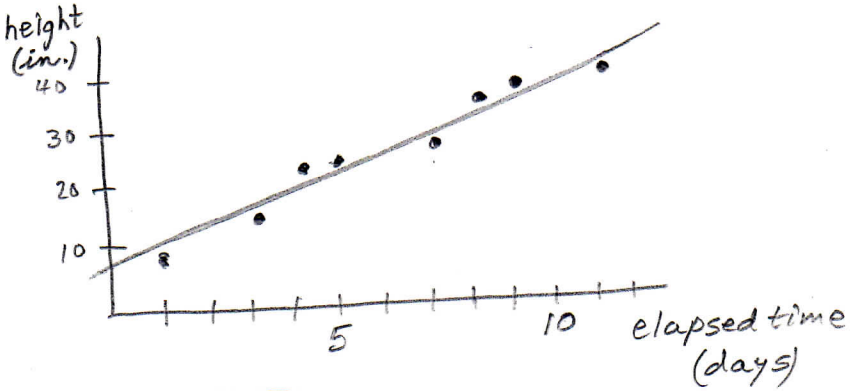
(c) By calc.: $\hat{y} = 7.3269... + 3.3205... x$

(d) $\hat{y}(6) \approx 7.3269 + 3.3205(6) \approx$ 27.25 in.

$\hat{y}(15) \approx 7.3269 + 3.3205(15) \approx$ 57.13 in.

(e) The estimate for 6 days (27.25 in.) uses <u>interpolation</u> and is probably quite accurate. However, the estimate for 15 days (57.13 in.) relies upon <u>extrapolation</u> beyond the domain of our data. We should not trust the estimate for 15 days unless we have a strong reason to believe that the generally linear trend will continue.

(f)



(g) slope $\approx$ 3.3205

units: inches/day

(h) The model predicts that for each additional elapsed day, the bamboo stalk will grow about 3.3205 more inches.