STAtistics / Mr. Hansen
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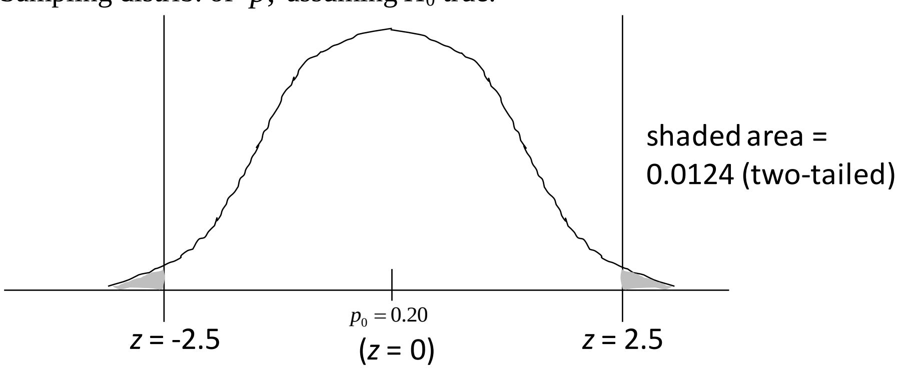
Solutions to #10.85 and #10.93

10.85 Let p = true proportion of white cars purchased in local metro area in 1993.
$H_0 : p = 0.20$
$H_a : p \neq 0.20$

<u>Assumptions for 1-prop. *z* test</u>
SRS? Not stated, but "random sample" was stated. ✓ Proceed with caution.
Is $n \le \frac{1}{10}N$? If n = 400, surely more than 4000 cars were bought in metro area in 1993. ✓
Is $np \ge 10$? $np \approx n\hat{p} = 400(\frac{100}{400}) = 100 >> 10$ ✓
Is $nq \ge 10$? $nq \approx n\hat{q} = 400(\frac{300}{400}) = 300 >> 10$ ✓

Sampling distrib. of $\hat{p}$, assuming H_0 true:



Test statistic: $z = \dfrac{\hat{p} - p_0}{\text{s.e.}} = \dfrac{\frac{100}{400} - 0.20}{\sqrt{\dfrac{pq}{n}}} \approx \dfrac{0.05}{\sqrt{\dfrac{(0.2)(0.8)}{400}}} = 2.5$

P-value = 0.0124 (two-tailed)
Since $P = 0.0124 < \alpha = 0.05$, we reject H_0.

Conclusion: Since $P < \alpha$, there is good evidence ($\hat{p}$ = 0.25, z = 2.5, P = 0.0124) that the true proportion of white vehicles sold in the local metro area in 1993 differs from the national proportion of 20%.

If α were 0.01, our <u>conclusion would change</u>. Since $P = 0.0124 > 0.01$, we would *not* reject H_0 for this new value of α. [In other words, we would say that there is no evidence ($\hat{p}$ = 0.25, z = 2.5, P = 0.0124) that the true proportion of white vehicles sold in the local metro area in 1993 differs from the national proportion of 20%.]

10.93 Let μ = true mean time (minutes) to achieve 100°F.

$H_0 : \mu = 15$

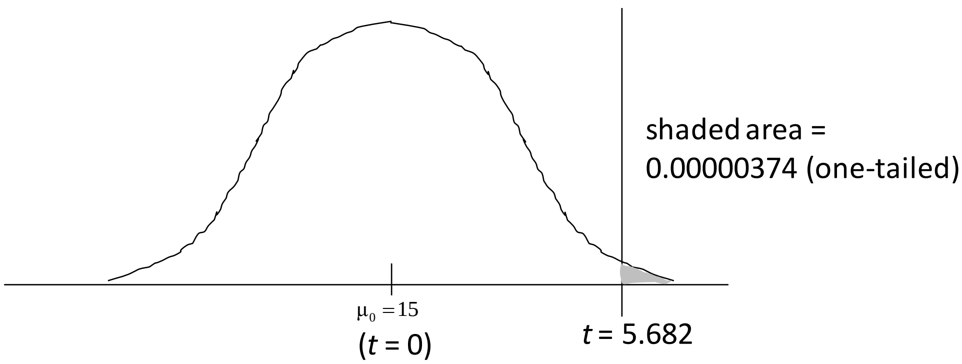
$H_a : \mu > 15$

<u>Assumptions for 1-sample *t* test</u>

SRS? Not stated, but "random sample" was stated. ✓ Proceed with caution.

Pop. distrib. normal? Not stated. However, n = 25, which is large enough in the absence of outliers or strong skewness. ✓ Proceed with caution.

Sampling distrib. of $\bar{x}$, assuming H_0 true:



Test statistic: $t = \dfrac{\bar{x} - \mu_0}{\text{s.e.}} = \dfrac{17.5 - 15}{\frac{s}{\sqrt{n}}} = \dfrac{2.5}{\left(\frac{2.2}{\sqrt{25}}\right)} = 5.682$

P-value = 0.00000374 (one-tailed)

Since $P \approx 0 < \alpha = 0.05$, we reject H_0. [In fact, we would reject H_0 for virtually any value of α since the result is so highly significant.]

Conclusion: Since $P < \alpha$, there is extremely strong evidence ($\bar{x}$ = 17.5, t = 5.682, df = 24, P = 0.00000374) that the true mean time to heat tubs to 100°F exceeds 15 minutes.