AP Statistics / Mr. Hansen
12/21/2011

Name: KEY
Mr. Hansen's use only (bonus point for spare batteries): _______

Big Quiz (60 pts.)

- Calculator is OK throughout.
- If a blank is provided, give the short answer that fits best.
- If a gap is provided, provide adequate justification/explanation, and circle your answer.

1. Identify each of the following random variables as discrete or continuous. You do not need to write the questions for #1, but you *do* need to write the word "discrete" or "continuous" each time in order to practice your spelling.

2 (a) age (to the nearest year) of a randomly selected citizen of Maine discrete
2 (b) diameter of a cylindrical table leg turned on a lathe continuous
2 (c) snowfall (to the nearest quarter inch) at a randomly selected location on earth on a randomly selected day discrete
2 (d) weight of a randomly chosen student continuous

2. Mr. Hansen's brother is a consistent 90% free-throw shooter. Mr. Hansen's brother is unflappable, and this probability never changes. Let X be the number of free throws made in 12 tries.
(a) Explain why X is a binomial random variable. Be complete.

2 1. Only 2 possible outcomes per trial, success or failure
2 2. Indep. trials; p never changes
2 3. $n = (\#\text{ of trials})$ is fixed
2 4. r.v. counts # of successes in n trials

(b) List the sample space of possible outcomes for X, along with their associated probabilities.

6

X	0	1	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	10^{-12}	10^{-10}	$5 \cdot 10^{-9}$	10^{-7}	$3 \cdot 10^{-6}$	$5 \cdot 10^{-5}$	.0005	.0038	.0213	.0852	.2301	.3766	.2824

2 (c) The set of all possible outcomes for a random variable, along with their associated probabilities (or probability
1+1+1 densities, in the case of a continuous r.v.) is called a distribution (*hint:* starts with the letter
1+1+1 D) and is usually depicted by means of a relative frequency histogram (*hint:* letters R, F, H) or a density curve. We use the former depiction for discrete random variables, the latter depiction for continuous random variables.

(d) Compute the probability that fewer than 8 free throws are made in 12 tries.

1+2 $P(X<8) = 1 - P(X \geq 8) = 1 - (.0213 + .0852 + .2301 + .3766 + .2824) = .004$

(e) Compute the probability that exactly 9 or 10 free throws are made in 12 tries.

1+2 $P(X=9 \text{ or } X=10) = P(X=9) + P(X=10) = .0852 + .2301 = .315$

ALT. METHOD: $P(X \leq 10) - P(X \leq 8) = .3409977 - .0256 = .315$

3. There are about 600,000 people in Washington, DC, of whom approximately 2% are HIV-positive. If you shake

the hands of randomly chosen DC residents, one by one, let Y denote the number of hands you need to shake in order to encounter an HIV-positive individual.

(a) Explain why Y is *not* a geometric random variable.

2 The geometric model requires indep. trials, i.e., sampling with replacement.

(b) Is it acceptable to treat Y as if it were geometric? Why or why not?

2 Yes, because the population is large compared to any realistic sample.

(c) Compute the mean of Y.

1+2 $\mu_Y = \frac{1}{p} = \frac{1}{.02} = 50$

(e) Compute the probability that the first HIV-positive person whose hand you shake is person 35 or later.

1+2 $P(Y \geq 35) = q^{34} = .98^{34} = .503$

(f) Compute the probability that the first HIV-positive person whose hand you shake is strictly *after* person 15 but strictly *before* person 79.

1+2 $P(15 < Y < 79) = P(16 \leq Y \leq 78) = P(Y \leq 78) - P(Y \leq 15)$

$= .79316 - .26143 = .532$

4. Mr. Hansen's latest SAT alternative test (the HAT, or Hansen Aptitude Test) has a mean of 540 and a s.d. of 110. Scores are approximately normally distributed. Compute

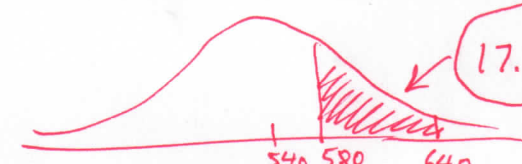
(a) the 60th percentile of the HAT

1+2

568 by calc.

(b) the proportion of test takers who score between 580 and 640

1+2



17.6% by calc.

(c) the HAT score that corresponds to an SAT score of 750, given that the SAT is approximately $N(500,100)$

1+2 SAT 750 is 2.5 s.d.'s above mean $(z = 2.5)$

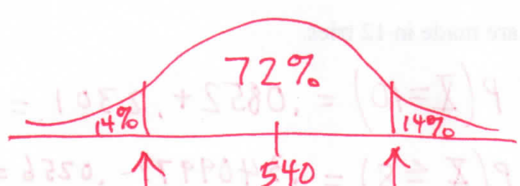
$\Rightarrow 2.5 = \frac{\text{HAT score} - \mu}{\sigma} = \frac{\text{HAT score} - 540}{110}$

$\Rightarrow 275 = \text{HAT score} - 540$

$\Rightarrow \text{HAT score} = 815$

(d) the cutoff HAT scores that capture the central 72% of the distribution.

1+2



$z = -1.080$ $z = 1.080$ by calc.

$540 \pm 1.08(110)$

$= 540 \pm 118.835$

or 421 to 659