AP Statistics / Mr. Hansen
11/15/2011

Name: KEY
Mr. Hansen's use only (bonus point for spare batteries): ______

Test on Probability and Transformations

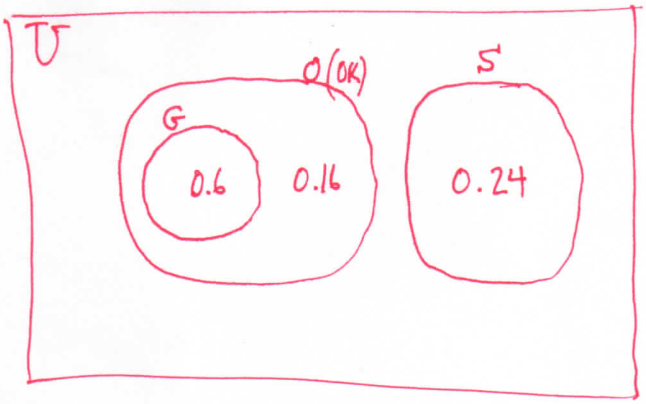
- Calculator is OK throughout.
- If a blank is provided, give the short answer that fits best.
- If a gap is provided, provide adequate justification/explanation, and circle your answer.

5
5
5

1. We will draw 2 cards, without replacement, from a well-shuffled deck. Let A be the event that a 7 is drawn on the first draw, and let B be the event that an 8 is drawn on the second draw. Are A and B mutually exclusive? (Write "yes" or "no.") NO Are A and B independent? NO
   Compute $P(A \cap B)$, correct to 6 decimal places: 0.006033 $= \frac{4}{52} \cdot \frac{4}{51}$

2. In the 2006 Physics Egg Drop Competition, conducted by Dr. Morse, the probability of a completely unscathed egg was 0.6, and the probability of an egg being unscathed or only cracked was 0.76. All other eggs were smashed. Let U be the universe of all eggs that were dropped. Draw a Venn diagram to illustrate the universe, as well as the relationship among events G (good drop), O (OK drop with no more than a crack), and S (smashed). Use the blank region below.

5



Also state the following to 3 decimal places of accuracy:

15 (3 ea.)

$P(O \cap \sim G) =$ 0.160
$P(G \mid O) =$ 0.789 $= \frac{.6}{.76}$
$P(O \mid G) =$ 1.000
$P(S) =$ 0.240
$P(S \cup G) =$ 0.840

Explain carefully why S and G are mutually exclusive but not independent. Label the probabilities that you use in your proof.

5 $P(S \cap G) = 0$ (since no overlap) $\Rightarrow$ mutually exclusive $\square$

5 $P(S \mid G) = 0$, but $P(S) = 0.24$; since these are unequal, S and G are not independent $\square$

OR... $P(S \cap G) = 0$, while $P(S) \cdot P(G) = 0.24(0.6) \neq 0$; since $P(S \cap G) \neq P(S)P(G)$, S and G are not independent $\square$

3. Two fair dice are rolled. Compute the probability of an even sum, given that neither die is a 1. Show your work (no credit without work).

	1	2	3	4	5	6
1						
2		4	·	6	·	8
3		·	6	·	8	·
4		6	·	8	·	10
5		·	8	·	10	·
6		8	·	10	·	12

5

$\frac{\text{13 winners}}{\text{25 possibilities}} = 0.52$

4. I feel lucky, and I think I can roll snake eyes (double 1) on my next roll of the dice. You offer to bet me with payout odds of 35:1. Is this a fair game? YES ~~Compute the expected value of the game for each dollar that I wager. (Work is needed for credit~~.)

2

EXPECTED VALUE COMPUTATION (omitted on 11/15/2011):

Let X = payout from one play of the game to Mr. Hansen

$$E(X) = \mu_X = \sum x_i p_i = (-\$1)\left(\frac{35}{36}\right) + (\$35)\left(\frac{1}{36}\right) = \boxed{\$0}$$

The game is fair, since the expected payout to Mr. Hansen is zero.

6. Write a simulation methodology to address the following question. Do not actually solve the problem.

An airplane has 100 passenger seats. However, no obese person can board if 2 or more obese people are already seated on that side of the fuselage. The fuselage has 2 sides (left and right, with 50 seats on each side). All 100 seats are reserved and assigned to passengers, 10% of whom are obese. We wish to know the probability that the third obese passenger to attempt to board will be denied boarding.

10

1. Designate 00-09 as "obese," 10-99 as "other passengers."
2. Draw digits 2 at a time from a random digit table. Ignore any pair that has already been used for the current boarding.
3. Note where the third obese passenger is placed, given that the entire list of 00-99 was previously sorted randomly, with the first 50 entries designated "left" and the other 50 designated "right."
4. If the third obese passenger cannot be seated (because of being on the same side as the first two obese passengers), write "SUCCESS." If the third obese passenger can be seated, write "FAILURE."
5. Re-randomize the left/right designations by sorting all 100 entries (00-99) as in step 3.
6. Repeat steps 2-5 a total of, say, 500 times.
7. Compute $\hat{p}$ as $\frac{\text{\# of successes}}{500}$. This is the probability estimate.

7. Again, if 10% of airline passengers are obese, compute the probability that an SRS of 9 (from a large pool) will include at least one obese person. Then explain why your answer would be different (1 short sentence) if the pool consisted of only the 100 passengers in #6.

5 $P(\text{at least one obese}) = 1 - P(\text{no obese}) = 1 - (.9)^9$

$= .613$

2 This assumes that $q = P(\text{not obese})$ is fixed at 0.9. If the pool were smaller, $P(\text{not obese})$ would decrease with each additional non-obese person chosen.

X 8. State p and q in #7. (Write equations.) p = 0.1, q = 0.9

9. A screening test for yawnitis is 99% sensitive (i.e., $P(\text{pos.} \mid \text{infected}) = 0.99$) and 97% ~~selective~~ specific (i.e., $P(\text{neg.} \mid \text{not infected}) = 0.97$). Compute the PPV of the test if yawnitis affects 2% of all students.

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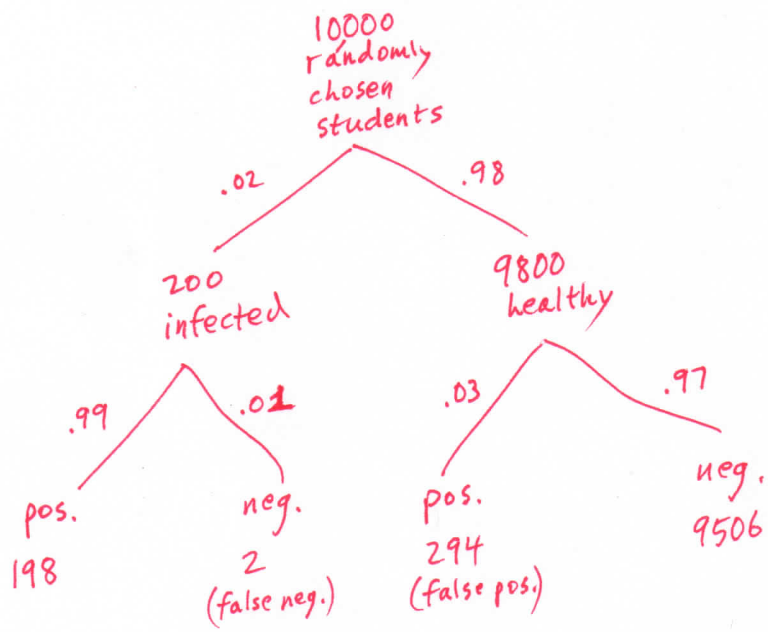

$$PPV = P(\text{infected} \mid \text{pos. reading}) = \frac{\text{true positives}}{\text{all positives}} = \frac{198}{198 + 294} = 0.402$$

2 10. PPV stands for positive predictive value and means

2 $P(\text{infection} \mid \text{positive reading})$.