Honors AP Calculus / Mr. Hansen
2/4/2011

Name: KEY
Bonus (for Mr. Hansen's use only): ________

Test through Chapter 8 (Calculator Required)

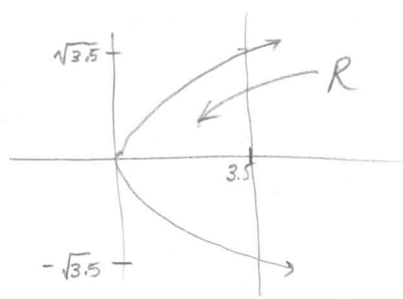
Rules

- You may not write calculator notation anywhere unless you cross it out. For example, fnInt(X^2,X,1,2) is not allowed; write $\int_1^2 x^2 dx$ instead.
- Adequate justification is required for free-response questions.
- All final answers in free-response portions should be *circled or boxed.*
- Decimal approximations must be correct to at least 3 places after the decimal point.

1. Let R be the region bounded by the parabola $x = y^2$ and the line $x = 3.5$.

(a) Sketch R.

(3)



(b) Compute the perimeter of R. Be sure to include the flat part.

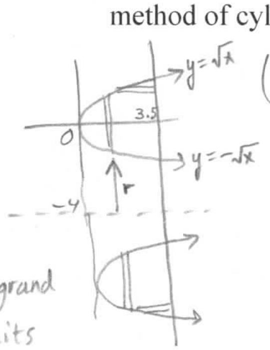
(4) integrand
(2) limits
(3) flat part
(2) final answer

Parameterized by $\begin{cases} x = t^2 \\ y = t \end{cases}$

$$L = \int_{-\sqrt{3.5}}^{\sqrt{3.5}} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt + 2\sqrt{3.5} = \int_{-\sqrt{3.5}}^{\sqrt{3.5}} \sqrt{(2t)^2 + 1^2}\, dt + 2\sqrt{3.5}$$

$$\approx 12.002$$

(c, d) Compute the volume of the solid of revolution formed when R is revolved about the line $y = -4$. Use (c) the method of plane slicing (planes perpendicular to the x-axis) and (d) the method of cylindrical shells. Use reverse side if necessary.



(c) $V = \int_0^{3.5} (\pi R^2 - \pi r^2)\, dx = \pi \int_0^{3.5} \left[(\sqrt{x} + 4)^2 - (-\sqrt{x} + 4)^2 \right] dx$

≈ 219.422 cu. units

(8) integrand
(3) limits
(2) answer

(d) $V = \int_{-\sqrt{3.5}}^{\sqrt{3.5}} 2\pi r h\, dy = \int_{-\sqrt{3.5}}^{\sqrt{3.5}} 2\pi (y+4)(3.5 - y^2)\, dy$

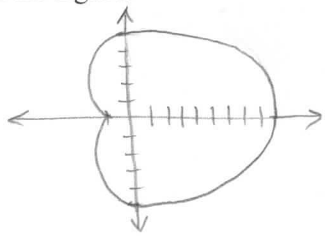
≈ 219.422 cu. units

(for each)

2. Consider the region in Quadrants I and II bounded by the limaçon defined by the polar equation $r = 5 + 4\cos\theta$.

(a) Sketch the figure.

(5)



- General shape
- (9,0) clearly shown
- (0,5) " "
- (0,-5) " "
- (-1,0) " "

(b) Compute the perimeter of the figure. Show work. Do not forget to include the flat part.

(6) integrand
(2) limits
(2) answer

$$L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{(5+4\cos\theta)^2 + (-4\sin\theta)^2}\, d\theta$$

$$\approx 36.688$$

or 28.344 for upper half with segment from (-1,0) to (9,0)

(c) Compute the area of the figure. Show work.

(6) integrand
(2) limits
(2) answer

$$A = \int_0^{2\pi} \frac{1}{2} r^2\, d\theta = \int_0^{2\pi} \frac{1}{2}(5+4\cos\theta)^2\, d\theta$$

$$\approx 103.673 \text{ sq. units}$$

3. It is 8:00 p.m., and Stu Dent has only 12 hours remaining before his 8:00 a.m. HappyCal final exam. He needs to sleep, of course, and any time he does not spend sleeping will be spent in a last-ditch and largely futile effort to cram knowledge into his brain (what some people refer to as "studying"). The question is, How should Stu allocate his time so as to maximize his expected score on the HappyCal final exam?

Stu has an accurate model of expected exam score, E, as a function of time spent studying and time spent sleeping. If x denotes the number of hours spent studying, the model predicts a contribution of $0.24\ln(x+1)-\dfrac{x^2}{500}$ points toward the exam score. By contrast, the model predicts a contribution of $\dfrac{0.6}{1+14e^{-0.6x}}-0.1$ points associated with x hours of sleep. The total predicted exam score, E, is the sum of the points contributed by studying and sleeping.

(a) If x denotes the number of hours spent sleeping, what is the domain for $E(x)$? Assume that Stu is a dorm student who could literally stumble out of bed and make it to his exam on time if he spent the full 12 hours sleeping.

(3)

$$\mathcal{D}_E = [0,12]$$

(b) If x denotes the number of hours spent sleeping, write an expression involving x for the number of hours spent studying.

(3)

$$12-x$$

(c) Write E as a function of a single independent variable. Be sure to define what your independent variable means.

Let x = # of hrs. of sleep, $12-x$ = # of hrs. of study

(4)

$$E(x) = \frac{0.6}{1+14e^{-0.6x}} - 0.1 + 0.24\ln(12-x+1) - \frac{(12-x)^2}{500}$$

(d) Write an algebraic expression for $E'(x)$.

(8)

$$E'(x) = -0.6\left(1+14e^{-0.6x}\right)^{-2}\left(14e^{-0.6x}\right)(-0.6) + \frac{0.24}{13-x}(-1) - \frac{2(12-x)}{500}(-1)$$

(14) (e) Find the number of hours of sleep, x, that would maximize Stu's expected exam score. Justify your answer. Use reverse side, please, but note that there is one more question.

(2 pts. per crit. pt or endpoint, 2 pts. per fcn. value, 2 pts. for answer)

(f) By how many points would Stu's expected exam score decrease if he slept for a full glorious 10 hours instead of the number of hours you wrote for part (e)? If you could not get an answer for part (e), simply state $E(10)$. Otherwise, compute the difference between $E(10)$ and $E(x_{optimal})$.

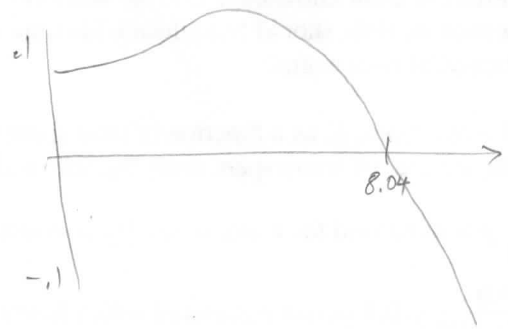
(3)

$E(10) \approx .735544$

$E(8.04586\ldots) \approx .792308$

Score drops by about 0.057 points.

E' has this graph:



Use root finder to find all crit. pts. for $E(x)$ on $[0,12]$:

$x \approx 8.04586046$ gives the only crit. pt.

$\therefore$ Check $E(x)$ at all endpts. and crit. pts.

x	$E(x)$	min.	max.
0	.2675...		
8.04586	.792308...		✓
12	.49379...		

Answer: 8.046 hrs. of sleep maximizes $E(x)$.